An Overview of Research on Teaching and Learning Mathematics

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Foreword

The Committee for Educational Science at the Swedish Research Council began its activities in March 2001. Its assignment is to promote research of high scientific quality with relevance for teacher education and pedagogical professional work. This means research on learning, knowledge formation, education and instruction. Like the rest of the Research Council, the Committee is responsible for research policy and research information.

The Committee distributes funds to research. In addition, it supports networks of researchers, arranges conferences and provides travel grants to stimulate international exchange among researchers. The Committee has initiated various reviews and surveys as well.

In order to further discussion about the field of educational science and its continued development, the Committee has asked some researchers to illuminate various themes connected with the Committee's assignment.

In this report Professor Rudolf Strässer, Luleå University of Technology, describes Swedish research on didactics of mathematics and how it relates to international research. The report also summarises Swedish doctoral and licentiate dissertations in the field, and gives an overview of Swedish institutions with activities in mathematics and pedagogy/education.

Stockholm, November 2004

Tjia Torpe Chairman	Ulf P. Lundgren Secretary General

Structure

The commission from Vetenskapsrådet – on terminology

In November 2003, the Committee for Educational Sciences of the Swedish Research Council (Vetenskapsrådets Utbildningsvetenskapliga kommitté) commissioned a report (forskningöversikt) on research on teaching and learning mathematics (forskning om lärande i matematik). UVK of Vetenskapsrådet wanted to have a map of ongoing national and international research in the area of teaching and learning mathematics[1].

Translating 'lärande' as 'teaching and learning' already points to the fact that a workable delimitation of the area to be described is in itself a complicated issue. In addition to this, the English and international terminology in the field is not homogeneous. In Anglo-Saxon countries (especially the United Kingdom and the USA), the field is most often described as 'research in mathematics education', while other countries (mostly European, especially France and Germany) prefer 'Didactics of Mathematics' (in France: 'Didactique des Mathématiques', in Germany: 'Mathematikdidaktik') even when publishing in English. Part of the background seems to be different histories of the word 'didactic'. In the English-speaking countries, 'Didactics' ended up as "literature or other art, intended to convey instruction and information. The word is often used to refer to texts that are overburdened with instructive or factual matter to the exclusion of graceful and pleasing detail so that they are pompously dull and erudite" (Encyclopaedia Britannica 2001, keyword 'didactic'[2]). As a consequence, for a person grown up with English as the working language, 'didactical' and 'didactics' will be negatively coloured. In contrast to this and especially for persons with a German educational background, 'didactics' seems to be more directly linked to Amos Comenius and his "Didactica Opera Omnia", which – together with his "Orbis Sensualium Pictus", the "forerunner of the illustrated schoolbook of later times" (see the Encyclopaedia Britannica again) – is often

[1] In Swedish, the mission was: "UVK finner det angeläget att få till stånd en kartläggning av den forskning som pågar, nationellt och internationellt, inom området 'lärande i matematik'".

[2] According to the same source, the "New Oxford Dictionary of English" offers "intended to teach, particularly in having moral instruction as an ulterior motive: *a didactic novel that sets out to expose social injustice"* and "in the manner of a teacher, particularly so as to treat someone in a patronizing way" (italics in original) as a definition of 'didactic'.

regarded as the start of textbook writing for general education and as the first attempt to create an educational system for the majority if not for everyone (a more detailed discussion of the recent terminological background and debate on 'didactics' is given by Björkqvist 2003, pp. 10–12).

In the present report, I will not go into details on the above controversy, but try to start from an inclusive understanding of the field (see section below) – not least because this offers the best possibilities to take into account what is happening in a comparably young research arena like Didactics of Mathematics in Sweden. When talking about research, I will use 'Didactics of Mathematics'. 'Mathematics education' will be used whenever I discuss issues in the actual teaching and learning of mathematics, including discussions about the school or university system and the role which mathematics plays in it.

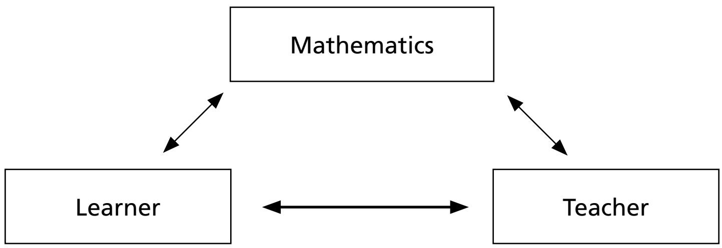
How to understand "research on teaching and learning mathematics"

Apart from the terminological problems already discussed in section 0, it is far from obvious how to understand research on teaching and learning mathematics. There is no agreed definition of what research in mathematics education (as the Anglo-Saxon 'world' would phrase it) or Didactics of Mathematics is about. It is even not easy to find an appropriate description of Mathematics, which obviously plays a major role in an understanding of Didactics of Mathematics. While Mathematics is undoubtedly one of the oldest scientific activities, and this discipline has a history of several thousand years, mathematicians still disagree on their subject. The fierce debates in the 1930s, or the controversies at the beginning of the Bourbaki description of Mathematics, clearly show that Mathematics in itself cannot be easily described even if defined. I will not go deeper into this fascinating issue of a meta-theory of the discipline Mathematics. Nevertheless it seems fair to start from a preliminary definition of Mathematics such as the "disciplinary analysis of (formal) patterns and structures". This is the essence of a rather unquestioned, but formal, description by Curry (1970) – even though this description overemphasises the product of the scientific activities in Mathematics, downplaying the procedural aspects of Mathematics.

With such a description of Mathematics, I start from the following description of Didactics of Mathematics: Didactics of Mathematics is made up of the scientific activities of describing, analysing and better understanding people's struggle for and with Mathematics. Sometimes this struggle is highly organised – for instance in compulsory schools or university departments of mathematics. Various sorts of organisations (e.g. journals and professional organisations) and standards (e.g. government regulations) play a specific role in this struggle.

In this report, I will use this description for identifying Didactics of Mathematics, which immediately shows that Didactics of Mathematics looks into the relation between human beings and a certain, very special type of reality, namely (formal) patterns and structures. The description given above implies that Didactics of Mathematics is a human science dealing with human beings. It also implies that the object with which these human beings struggle is not material in the sense of being directly touchable. When learning and/or teaching mathematics, human beings first have to create representations of the patterns and structures in order to study the relations ‘embodied’ in the representations. This has consequences for the scientific analysis of these activities.

In modern societies, the relation between human beings and Mathematics often unfolds into a relation between **three** agents in the human struggle for/with Mathematics: Mathematics itself, the teacher and the learner. This is the so-called ‘didactical triangle’ (see also the diagram below). The difference between the two human agents (teacher and learner) is usually seen in the fact that the teacher should know more about the object of learning (i.e. mathematics) than the learner knows.



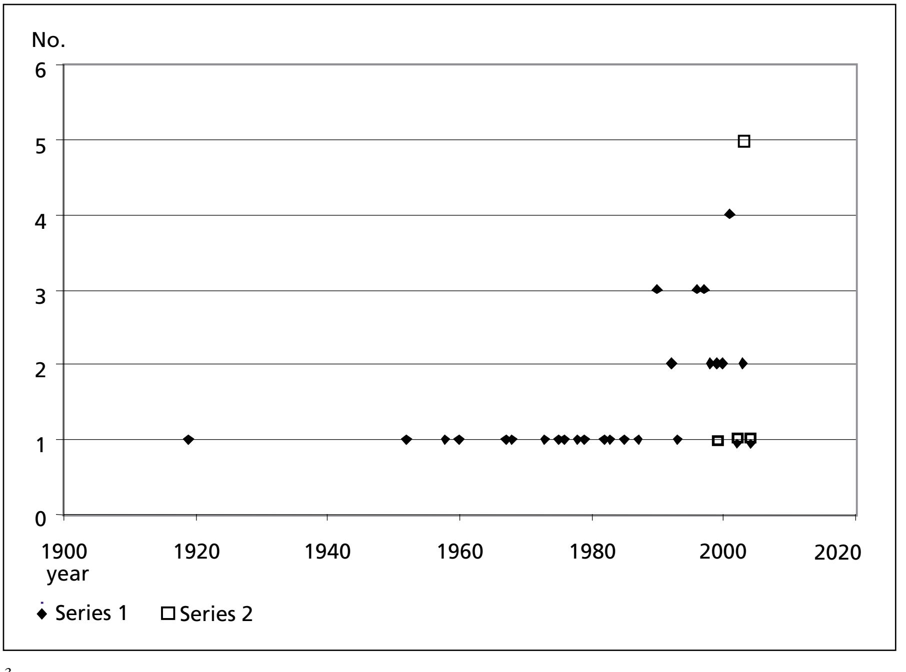
One should mention that the didactical triangle is only a model in the strict sense: it does not take into account the “environment” of a “didactical system”, such as parents, school administration, professional teacher organisations, mathematicians interested in education, institutional constraints, society, history and many other issues (for a more detailed analysis see Chevallard 1985/91). We will see in the report that Didactics of Mathematics – also in Sweden – does analyse more than the didactical triangle. For the moment, the theoretical analysis will be ended here. The next sections are devoted to a factual description of what is going on in Didactics of Mathematics in Sweden and internationally.

Swedish research on teaching and learning mathematics

Places in academia

In order to describe Swedish research on teaching and learning mathematics, we start with a very simple approach. We searched the websites of all Swedish universities and university colleges (for a list of them see the respective website[3]) and other available sources (especially Engström 1999) for dissertations and licentiate theses on Didactics of Mathematics. This produced a list of 41 Ph.D. dissertations and 8 licentiates (see Appendix A) – with abstracts for the majority – giving an overview of where, and by whom, research is done, on what, within Didactics of Mathematics in Sweden. Condensing this information into a graph offers the following picture of "Dissertations and Licentiates over time" (see Figure 1).

Figure 1. "Series 1" shows the dissertations and "Series 2" the licentiates, with the two marks of "Series 2" for years 2002 and 2004 obscuring the respective marks of "Series 1".



[3] http://katalogen.sunet.se/kat/education/universities

Table 1. Swedish dissertations (Ph.D. theses)

Author	**Year**	**Place**	**Keywords** (for an explanation see section on Swedish topics.)
Johnsson	1919	Uppsala	problem-solving
Wictorin	1952	Göteborg	
Werdelin	1958	Lund	theories of learning
Dahllöf	1960	Stockholm	*curriculum*
Postlethwaite	1967	Stockholm	socio-cultural studies, *curriculum*
Ekman	1968	Uppsala	geometry
Larsson	1973	Lund	
Holmberg	1975	Lund	technology
Noonan	1976	Stockholm	socio-cultural studies
Håstad	1978	Uppsala	*curriculum*
Kristiansson	1979	Göteborg	*curriculum*
Allwood	1982	Göteborg	statistics, mental models, problem-solving
Warg	1983	Uppsala	statistics, mental models, problem-solving
Hellström	1985	Lund	teacher education
Neuman	1987	Göteborg	early numbers, mental models, *phenomenography*
Bergsten	1990	Linköping	mental models, language
Hedrén	1990	Linköping	early numbers, geometry, technology
Pettersson	1990	Stockholm	problem-solving
Ahlberg	1992	Göteborg	early numbers, problem-solving, mental models
Löthman	1992	Uppsala	rational numbers, problem-solving, *phenomenography*
Wyndham	1993	Linköping	problem-solving, socio-cultural studies
Chen	1996	Stockholm	socio-cultural studies
Dunkels	1996	Luleå	advanced mathematical thinking
Ekeblad	1996	Göteborg	early numbers, mental models, phenomenography
Engström	1997	Stockholm	rational numbers, mental models, theories of learning
Sandahl	1997	Linköping	technology, socio-cultural studies
Wikström	1997	Göteborg	mathematical modelling, advanced math. thinking
Dahland	1998	Göteborg	technology, teacher professional development
Åberg-Bengtsson	1998	Göteborg	visualisation, *phenomenography*
Hedenborg	1999	Stockholm	early numbers, mental models
Runesson	1999	Göteborg	rational numbers, *phenomenography*
Häggblom	2000	Åbo	early numbers
Lingefjärd	2000	Göteborg	mathematical modelling, technology
Bergqvist	2001	Umeå	advanced mathematical thinking
Emanuelsson	2001	Göteborg	*phenomenography*
Lithner	2001	Umeå	advanced math. thinking, problem-solving, mental models
Möllehed	2001	Lund	problem-solving, gender
Palm	2002	Umeå	mathematical modelling
Bentley	2003	Göteborg	teacher education
Samuelsson	2003	Uppsala	rational numbers, technology
Löwing	2004	Göteborg	teacher education, socio-cultural studies

The 41 dissertations have been defended between 1919 and 2004, while the 8 licentiates were more recently finished in the years 1999 to 2004. From the graph, it is obvious that research in Didactics of Mathematics in Sweden started to grow in the 1970s and has reached a certain rhythm and continuity since the 1990s. In addition to Ph.D. theses, licentiate theses appear at the end of the 1990s. If we compare this with the international developments (see the next section of this report), it seems fair to say that Sweden is a latecomer in the area of Didactics of Mathematics. In countries like France, Germany, the UK and the USA, this scientific discipline arose in the 1970s if not the late 1960s.

In terms of the local distribution of the dissertations in the whole of Sweden, one centre is conspicuous: 14 of the dissertations, roughly a third of the total, have been submitted in Göteborg (see table 1). No other university comes near to this number, the next being Stockholm with 7 (putting together all dissertations at the different places in Stockholm) and Uppsala University with 6 dissertations (including an early start in 1919). Linköping (4 dissertations), Lund (5 dissertations) and Umeå (3 dissertations) have also had more than one dissertation accepted.

Table 2. Swedish licentiates

Author	**Year**	**Place**	**Keywords** (for an explanation see section on Swedish topics.)
Engström	1999	Stockholm	geometry, technology
Bjerneby Häll	2002	Linköping	curriculum
Bremler	2003	Stockholm	advanced mathematical thinking, (technology)
Johansson	2003	Luleå	curriculum, (technology)
Nilsson	2003	Växjö	probability, mental models
Ryve	2003	Mälardalen	advanced math. thinking, mental models, language
Taflin	2003	Umeå	problem-solving
Juter	2004	Kristianstad/Luleå	advanced math. thinking, functions, mental models

For the licentiates, the local distribution is largely influenced by the fact that half of them (four out of eight) have actually been written within the framework of the graduate school sponsored by Riksbanken's Jubileumsfond and Vetenskapsrådet (see below, part on national institutions). Here, Luleå tekniska universitet (with 2 licentiates) stands out. Closer inspection shows that Stockholm's Lärarhögskola is another important place (not linked to the RJ/VR graduate school) because one student is from Stockholm, although the licentiate was submitted at South Bank University, London.

In order not to build only on dissertations for the regional distribution of places where research in Didactics of Mathematics is done, we can offer two other sources. In his situation report *(lägesbeskrivning)* on "Matematikdidaktiken i Sverige", Ole Björkqvist (2003, p. 21) named the following institutions as having a research education programme (*forskarutbildningsprogram*):

- Institutionen för matematik och naturvetenskap, **Kristianstad** (formally linked to Luleå)
- Matematiska institutionen, **Linköping**
- Institutionen för matematik, **Luleå**
- Lärarutbildningen, **Malmö**
- Matematiska institutionen, **Stockholm**
- Matematiska institutionen, **Umeå**
- Matematiska institutionen, **Uppsala**
- Matematiska och systemtekniska institutionen, **Växjö**

While Björkqvist has a longer list of research groups (cf. loc. cit., pp. 18–20), it seems fair to name only the places with a research programme – taking into account the comments Björkqvist himself made (loc. cit., p. 16). The most remarkable fact about this list is already mentioned in Björkqvist's report: most of the institutions having a research programme clearly identify themselves as Mathematics institutions, and there is only one institution (Lärarutbildningen in Malmö), which has an educational profile. In terms of institutionalisation, Didactics of Mathematics in Sweden in most places has its home in Mathematics as a scientific discipline; Education comes only second as a 'home' of Didactics of Mathematics in Sweden – although Göteborg is an excellent counterexample (for the Göteborg situation see below).

As an additional source, and in order to be as up-to-date as possible, we checked the Internet in April/May 2004, using the list of Swedish universities and university colleges already mentioned above[4]. We looked for indications of activities clearly linked to Didactics of Mathematics and found the following institutions reporting on Didactics of Mathematics using the Internet:

- University College of **Borås**
- University of **Gothenburg** / Chalmers University of Technology
- University of **Jönköping**
- University of **Kalmar**
- University of **Karlstad**
- **Kristianstad** University College
- University of **Linköping**
- University of **Luleå**
- **Malmö** University

[4] http://katalogen.sunet.se/kat/education/universities

- **Mälardalen** University College
- **Stockholm**: Institute of Education (Lärarhögskolan i Stockholm) / Royal Institute of Technology / Stockholm University
- **Umeå** University
- **Uppsala** University

(See Appendix B.) This list is a bit longer than the one from Björkqvist 2003 – with Kristianstad formally linked to Luleå and Mälardalen linked to Stockholm University. It shows some institutions which are not famous, but have a standing tradition in the field – such as Borås and Jönköping. The list also indicates places where Didactics of Mathematics as a research field is "under construction" – such as Kalmar, Karlstad, Malmö and Mälardalen. In 2004, Kalmar and Karlstad even announced a position in Didactics of Mathematics; Malmö will fill a position in Didactics of Mathematics during this year (personal communication from Malmö).

"Swedish" topics

The search for Swedish Ph.D. theses and licentiates is not only helpful in finding centres of research in Didactics of Mathematics. The theses can also be indicative of topics researched in Sweden. In order to find out about research topics, we used the classification of the International Group of Psychology of Mathematics Education (PME; for details see below) and – judging from the abstracts – tried to classify the Ph.D. theses and licentiates with these keywords. In order to adequately classify the dissertations, we added two categories: "curriculum" (for identifying work analysing the relationship of mathematical knowledge to political/pedagogical concepts and prescriptions; used for four Ph.D. theses and one licentiate) and "phenomenography" (as a typical research paradigm for Sweden; 6 dissertations were explicitly related to this approach in the title and/or abstract).

First we tried to assign an area within mathematics to each dissertation – and we were successful for 18 Ph.D. theses and 4 licentiates. In some sense, the five classifications with "curriculum" also can be counted in this category. To put it differently: about half of the dissertations have a clear link to specific mathematical topics – which strengthens the observation that mathematics is the "home" of Didactics of Mathematics in Sweden. In terms of a development over time, more global approaches (classified "curriculum") fade out (with the exception of the recent licentiate from Luleå) in favour of more specific analysis of delimited mathematical topics.

Assigning “problem-solving” (used for 9 Ph.D. theses and 1 licentiate) shows the high interest in the process-aspect of the human struggle with mathematics, while assigning “mental models” (9 Ph.D. theses / 3 licentiates) attests the attempt at understanding cognitive structures, which makes this process possible. Research into technological aspects of teaching and learning mathematics began as early as 1975 (6 Ph.D. theses / 3 licentiates). The classification of two licentiates as “technology” research should be commented upon: they look into the role of textbooks in mathematics education – and the classification as “technology” research indicates that it is not only ‘modern’ technological innovations, such as computers and (hopefully appropriate) software, which play a decisive role as teaching/learning aids in mathematics.

In his ‘*lägesbeskrivning*’, Björkqvist (2003, pp. 34–36) identified internationally known Swedish research in Didactics of Mathematics. He listed:
– gender issues (‘*genusfrågor*’)
– teaching/learning quality under special circumstances (‘*kvalitet i lärandet under speciella förhållanden*’[5])
– phenomenographical approach (‘*fenomenografiska analyser av uppfattningar inom matematiken*’)
– mathematics education and democracy (‘*matematikundervisning och demokrati*’)
– history of mathematics and mathematics education (‘*matematikens och matematikundervisningens historia*’[6])
– computer-aided learning & teaching (‘*datorstött lärande och datorstödd utvärdering*’)
– problem-solving at upper secondary level (‘*beteende vid matematiskt problemlösande i gymnasiet*’)
– understanding symbols and mathematical language (‘*symbolkänsla och förståelse av matematisk språk*’).
– new types of (national) assessment in mathematics[7] (‘*nya typer av (nationella) prov i matematik*’).

This list fits well with the earlier studies done by Engström 1989 and Bergsten 2002 – and it confirms the results of the analysis of the dissertations and the information gathered in Appendix B.

[5] We assume this is because of the “SUM” network based in Jönköping; see Appendix B.

[6] Not least because of the research done in Uppsala.

[7] Most probably because of the “PRIM” group and research done in Umeå.

Björkqvist (2003, p. 37) also identifies two research topics which are definitely missing in Sweden: research into specific topics ('Stoffdidaktik') and research into "how" Mathematics is taught / learned. At a first glance, the lack of 'Stoffdidaktik' seems to contradict what we learned from the analysis of the dissertations. A more detailed approach shows that the 22 (18+4) dissertations classified with a specific mathematical topic had a clear mathematical focus, but they were all (except perhaps the dissertations of Ekman and Häggblom) using this mathematical focus to study some other didactical issue. Their main interest was not in trying to find the best way to teach this special topic – which would be the case with a 'Stoffdidaktik' approach (at least if used with a narrow understanding of Stoffdidaktik; for a discussion of this approach, see Strässer 1996). This is particularly important as international experience shows that research into specific topics normally helps to maintain the links between mathematics as a scientific discipline and Didactics of Mathematics. Places and institutions which do not investigate specific mathematical topics such as algebra or geometry tend to lose contact with university mathematics.

The second omission (research into "how" Mathematics is taught / learned) is all the more astonishing as this is one of the home-grown research perspectives of Didactics of Mathematics. In addition, the reality of Swedish research in Didactics of Mathematics may be a little more complicated. The two recent Ph.D. dissertations by Bentley (2003) and Löwing (2004) can be seen as an indication of a changing research focus – and the major 'KULT' project in Göteborg/Uppsala looks into this very issue. Björkqvist could not take into account the two recent Ph.D. theses, and overlooked the KULT project because he concentrated on institutions linked to mathematics.

National institutions

A research domain is not only influenced by, and not only dependent on, individual researchers working in university departments or research institutes. The life and development of a scientific discipline are heavily influenced by institutions, which facilitate and structure the communication between the individual researchers, which in turn organises communication in a research community. This section will present and comment on scientific journals, conferences, scientific associations and other institutions, which facilitate and further the communication between the researchers in the university departments.

For Didactics of Mathematics in Sweden, three **journals** seem of special importance: the Swedish teacher journal *Nämnaren*, a 'Nordic' journal dedicated to research entitled *NOMAD*, and the *Medlemsbrev* of the Svensk Förening för MatematikDidaktisk Forskning (SMDF). We will briefly describe these journals.

Nämnaren is clearly dedicated to the mathematics teacher in school; its audience consists mainly of teachers. The vast majority of subscriptions comes from individual teachers and from schools as institutional subscribers. Nämnaren is run by the National Center for Mathematics Education (NCM) in Göteborg and "is aimed at teachers, teacher trainers, researchers and the staff responsible for basic education, further education and development work" (citation from the website[8]). In fact, the 'Nämnaren' project is more than a journal with four issues per year; it also includes activities like 'Nämnaren on the web' and 'NämnarenTEMA', which publishes books on topics of interest for Swedish mathematics education. As it is the only 'purely' Swedish journal in mathematics education and because of its wide audience within the teaching staff at schools, *Nämnaren* is most important for making Swedish research known to teachers all over the country. Nowadays, another journal, the *Nordisk Matematisk Tidskrift (normat)*, also co-edited and supported by NCM, is a Nordic (Denmark, Norway, Sweden) journal aiming at popularising mathematics and is mentioned for the sake of completeness here.

'*NOMAD*' (the official abbreviation for 'Nordic Studies in Mathematics Education') is a research journal with varying intervals of publication. For 2004, four issues are planned. On its homepage[9], one reads: "Nomad is a journal for research and developmental work in mathematics education. It addresses all who are interested in following the progress of this field in the Nordic countries, Denmark, Finland, Iceland, Norway and Sweden. The most important aim of the journal is to stimulate, support and foster Nordic researchers and post-graduate students in mathematics education and to develop mathematics teaching and teacher-education in theory and practice at all levels of the educational system. … The editors welcome articles about reports and surveys of research and development works, discussions of basic questions in mathematics education, theoretical analyses and empirical studies. We also welcome brief reports of research, research literature, and conferences, critiques of articles and book reviews." From this text, it is obvious that NOMAD sees itself as a journal with an audience dedicated to research. At present, Ole Björkqvist from Finland is the scientific editor; Johan Häggström from NCM, Göteborg, acts as managing editor.

8 http://www.ncm.gu.se/index.php?name=Namnaren-project

9 http://www.ncm.gu.se/index.php?name=nomad-bakgrund_eng

The *Medlemsbrev* of the Svensk Förening för MatematikDidaktisk Forskning (SMDF; two issues per year) should also be mentioned as a means of communication in the Swedish research community of Didactics of Mathematics. The last issues even started to print longer papers, which could slowly turn this more or less informal publication into an important link between Swedish researchers in Didactics of Mathematics. While the 'Medlemsbrev' of the Swedish Mathematics Society (SMS) also has papers related to teaching and learning mathematics in Sweden, it is different from the SMDF *Medlemsbrev* insofar as most of the texts presented in the former are more or less critical toward Didactics of Mathematics as a scientific discipline. It should nevertheless be mentioned as a forum where issues of Swedish Didactics of Mathematics are discussed. Judging from the last two issues of the SMS 'Medlemsbrev' and the way in which the work of the 'Matematik-Delegation' (for the 'Delegation' itself see end of this section) was commented upon, one gets the impression that inside SMS quite different, sometimes even contradictory voices are heard in relation to Didactics of Mathematics.

In addition, one should not forget the vast variety of scientific journals in education/pedagogy. They also tend to publish papers on teaching and learning mathematics – and often have a long-standing tradition and a well-developed system of quality control by peer review.

Conferences are another important means of communication in a research community – and there is at least one conference special to Swedish research in Didactics of Mathematics. Every second year and in close local cooperation with *Matematikbiennalen*, the Svensk Förening för MatematikDidaktisk Forskning (SMDF) organises a conference, the 'Swedish Mathematics Education Research Seminar' (in Swedish: *MAtematikDIdaktiska Forskningsseminariet*, abbreviated 'MADIF'). Activities comprise invited plenary sessions (partly with speakers from outside Sweden), paper sessions where participants present their own work, and a plenary panel on the topic of the conference title. The fourth MADIF conference was held in Malmö in 2004. The theme of the conference was "Mathematics and Language" and it attracted more than 100 participants mainly from Sweden[10]. *Matematikbiennalen* itself is a huge effort (normally with more than 3,000 participants) "to improve Education in Mathematics … a lot of programs and exhibits give ideas on how to make improvements in teaching of Mathematics. The Conference Program hopefully attracts everyone who works with Mathematics. It consists of lectures, seminars, workshops, posters

10 For more information see http://www.mai.liu.se/~chber/SMDF/mad4eng.htm.

and exhibitions" (cited from Matematikbiennalen's homepage[11]). An optimistic approach (like the one taken by Ole Björkqvist 2003) regards these conferences and other indicators as proof that the "proportion of teachers familiar with modern research in mathematics education is likely higher in Sweden than in most other countries" (see loc. cit., p. 35; translated into English and repeated in Emanuelsson & Johansson 2004, p. 8).

At least two other major '**institutions**' within Swedish Didactics of Mathematics should be mentioned. From 2001 until 2006, Riksbankens Jubileumsfond is sponsoring a national **graduate school**, at which 20 Ph.D. students in different universities or university colleges (Göteborg, Kristianstad, KTH, Linköping, Luleå, Mälardalen, Stockholm University, Umeå, Uppsala, Växjö) do research on "*matematik med ämnesdidaktisk inriktning*". The graduate school is also supported by Vetenskapsrådet and organises national courses/seminars. So far, the graduate school has 'produced' half of the Swedish licentiates[12]. Leder et al. (2004) give a concise description of this most important Swedish institution in Didactics of Mathematics. Since the beginning of 2003, a NORFA graduate school complements this activity (see below). In addition, but not in direct connection with the graduate school, a network '*Forskning om lärande i matematik, naturvetenskap och teknik*' is under construction, with Inger Wistedt from Pedagogiska Institutionen at Stockholm University as key person. This project has a very explicit interdisciplinary approach and aim (cf. Wistedt 2004).

Secondly, in Göteborg and linked to Göteborg University, there is a **national centre** for Didactics of Mathematics, 'National Center for Mathematics Education (NCM)' (in Swedish: *Nationellt Centrum för Matematikutbildning*). On January 1st, 1999, NCM was founded by the Swedish government as a national **resource centre** for mathematics education. Its most important activities seem to be the editing of the three journals *Nämnaren, normat* and *NOMAD*, the development of a national library and documentation on Didactics of Mathematics, and working as a clearinghouse and resource centre for teachers from pre-primary to university level[13]. One of the major recent activities of NCM was

[11] http://www.lut.mah.se/nms/matematik/biennal-04/MABI_03-11_eng.pdf

[12] For more information see the end of Appendix B and http://www.msi.vxu.se/Forskarskolan/.

[13] For details see the Göteborg part of Appendix B and http://www.ncm.gu.se/.

to act as secretary of the *Matematik-Delegationen*[14], which was an initiative of the government to support mathematics and mathematics education[15].

Additional remarks

Looking back on the description of the Swedish research community in Didactics of Mathematics – and somewhat informed by personal experience not limited to Sweden – I would like to make some general remarks on the Swedish situation.

As a first remark, I want to state explicitly that Swedish research in Didactics of Mathematics started rather late in comparison to other countries. In Sweden, Didactics of Mathematics really got off the ground in the 1990s, which seems to be fairly late in industrialised countries. With this late start, research in Didactics could profit from a developed international research scene as well as from the excellent Swedish research in Education/Pedagogy (at that time concentrated in Göteborg and Uppsala). As a consequence, the late start was somehow 'compensated' by stormy development, additionally supported by the creation of a national graduate school in the research domain.

As in other countries, Didactics of Mathematics can be institutionalised in the university departments of either Mathematics or Pedagogy/Education. In fact, in Sweden the area is hosted by Mathematics at the majority of universities. Both 'solutions' – Didactics of Mathematics within Mathematics or within Pedagogy/Education – have advantages and disadvantages. From the situation in Germany, where one can find examples of either means of institutionalisation, one learns that putting Didactics of Mathematics in the Mathematics Department normally strengthens the links of didactics with the discipline of Mathematics; research topics tend to be specific even for sub-domains of Mathematics, such as Geometry or Statistics, with a certain neglect of topics related to the early struggle with mathematics, for instance in primary schooling. In this institutional situation, demands on mathematical knowledge are relatively

14 See http://www.matematikdelegationen.gov.se/

15 From the mission of the *Matematik-Delegationen: "Delegationens uppdrag är att stärka matematikämnet och matematikundervisningen i hela utbildningssystemet, från förskola till högskola. Delegationen skall utgå från en analys av den nuvarande situationen och utarbeta handlingsplaner med förslag till åtgärder för att*

- *förbättra attityder till matematikämnet*
- *öka intresset för matematikämnet*
- *utveckla matematikundervisningen*
- *stimulera elever/studenter till fortsatta studier inom området"*

high, and genuine pedagogical questions tend to be downgraded in internal discussions inside such a research unit or department.

With the research in Didactics of Mathematics as part of a pedagogical, educational institution, the advantages often turn into disadvantages and vice versa. Questions deeply linked to specific issues of a mathematical sub-domain (for instance, the role of chance and probability in Statistics or the role of symbolic manipulation in Algebra) may not attract the special attention of colleagues from general pedagogy/education – even though they are often very pleased to exemplify their general results with the presumably easy and well-known subject of mathematics.

The situation in Göteborg indicates the consequences of this dilemma in a prototypical way. With strong Departments of Mathematics AND Pedagogy/ Education, and with an additional national centre like NCM working in the field of Didactics of Mathematics, the three 'players' (Mathematics / Pedagogy / NCM) compete and somehow do not come to grips with Didactics of Mathematics. At one of the strongest places in Mathematics (but partly openly hostile to Didactics) and surely *the* pedagogical/educational centre in Sweden, the best available and most differentiated structure in the whole country has not turned into a productive environment for Didactics of Mathematics. Although it is best equipped nationally in terms of scientific resources, Didactics of Mathematics seems not to grow in this place – and the students from the graduate school sponsored by the RJ fond and Vetenskapsrådet move away from Göteborg.

International research on teaching and learning mathematics

On places and topics

In 1908, the International Congress of Mathematicians in Rome marked the start of international scientific activity about teaching and learning mathematics. An international commission was founded – called 'Internationale Mathematik-Unterrichts-Kommission (IMUK)', which under the first president Felix Klein commissioned and published descriptions of national teaching of mathematics and cross-national comparisons (see the series 'Berichte und Mitteilungen, veranlasst durch die Internationale Mathematische Unterrichtskommission', Leipzig: Teubner). After a backlash of these activities during World War I and only limited efforts until the 1950s, Didactics of Mathematics grew considerably following the 'Sputnik shock', i.e. since the late 1950s. Here one could even mention a special 'Nordic' publication on mathematics education: a description of the teaching of mathematics to 'pupils up to the age of 16' in the Nordic countries, dated 1958 (see National Subcommissions 1958, published as a supplement of the *Nordisk matematisk tidskrift*; for the report on Sweden, see Sjöstedt 1958). Especially in the USA, huge sums of money were spent to further mathematics and science education, to close the gap (as it was seen at that time) between the technological developments in the Soviet Union and the USA. The Organisation for Economic Co-operation and Development (OECD)[16] then sponsored international seminars (for a summary see OECD 1961b; OECD 1961a is the documentation of a most influential OECD seminar of that time). Documents from these activities deeply influenced curriculum change all over the world, mainly opting for 'New Mathematics', which was often characterised by, if not identified with, teaching set theory in compulsory schools. This educational and curriculum change was also linked to a breakthrough of 'Bourbaki'-style Mathematics in European and US universities.

The International Mathematical Union (IMU) renewed its commission on education, now named the International Committee on Mathematics Instruc-

[16] For its website see http://www.oecd.org/.

tion (ICMI). In 1969, ICMI held its first international conference in Lyon, which was open to everyone interested in Mathematics Education. This congress started the series of international conferences now known as the International Congress of Mathematics Education (ICME), held every fourth year and with ICME-10 taking place in Copenhagen in July 2004 (the 10th congress in this series). The late 1960s and the 'student revolution' gave an additional push – for instance marked by the creation of the French 'Instituts de Recherche en Didactique des Mathématiques (IREM)' which tried to cope with the introduction and consequences of the so-called 'New-Math movement' and its specific French version, namely the hasty introduction of mathematics according to the Bourbaki style into teaching mathematics in compulsory schools. Taking an unusual (for France) local approach (every major university had its own IREM), the IREMs cooperated more or less loosely within a national board and national committees on specific topics like the history of mathematics or geometry. Different moves can be seen in Germany and the United Kingdom, which took a more centralised approach than France. Sponsored by major national companies, national research centres were created (in Germany the 'Institut für Didaktik der Mathematik – IDM' in Bielefeld; in the UK the 'Shell Centre' in Nottingham).

Nowadays, Didactics of Mathematics is well developed in central Europe (for instance in France, Germany, the UK) and in Israel and the USA. It seems fair to distinguish two well-established scientific communities: the Anglo-Saxon community, which publishes in English (with researchers mainly based in Israel, UK, USA and some Asian countries) and the French-communicating researchers (mainly from France, Greece, Spain and South America). Both communities somehow cooperate in the organisation 'Psychology of Mathematics Education (PME)', a group of researchers in Didactics of Mathematics, established in 1976 at ICME-3 in Karlsruhe. PME is linked to ICMI by formally being a subgroup affiliated with ICMI, but in fact working quite independently[17]. The PME group organises a conference every year to communicate and discuss about research within the field of Didactics of Mathematics. The first PME conference was held in 1977 in Utrecht/the Netherlands. In 2000 it was in Tokyo, 2001 in Utrecht/the Netherlands, 2002 in Norwich/UK, and 2003 in Honolulu/USA, to name only the latest. In 2004, PME-28 is being held in Bergen/Norway; in 2005, Melbourne/Australia will host the PME conference. According to my judgment, PME is the most important and inclusive international organization dealing with research in Didactics of Mathematics.

[17] Mmore information on PME can be found at http://igpme.org/

In order to structure and classify the hundreds of research reports offered and peer-reviewed for every conference, the PME community developed a list of research categories. At present, this list is under discussion and reconstruction, so I can only cite the latest list of research domains to be found in the proceedings of the 2002 conference in Norwich:

1 advanced mathematical thinking
2 affective factors
3 algebra
4 assessment
5 beliefs
6 technology
8 early numbers
9 epistemology
10 functions
11 gender
12 geometry
13 visualization
14 language
15 mathematical modelling
16 measurement
17 mental models
18 metacognition
19 proof
20 probability
21 problem- solving
22 rational numbers
23 socio- cultural studies
24 non-elementary numerical reasoning
25 teacher education and professional development
26 theories of learning
27 data handling
28 Other

(See PME-2002 proceedings; Cockburn & Nardi 2002, Vol. 1, pp. xliv-xlvii.)

This basically alphabetical list (some omissions of numbers already show ongoing changes in this classification) can be rearranged according to the 'didactical triangle' of mathematics – teacher – student, yielding the following more structured list:

Mathematics: advanced mathematical thinking, algebra, early numbers, functions, geometry, proof, probability, rational numbers, non-elementary numerical reasoning, and mathematical modelling.

Human actors (teacher & learner): affective factors, beliefs, gender, visualization, language, mental models, metacognition, problem-solving, theories of learning.

Teacher alone: teacher education & professional development.

System environment: assessment, technology (computer), socio-cultural studies.

Methodology: measurement, data handling.

Other: epistemology.

From this rearrangement, it seems fair to say that the PME community somewhat neglects researching the early years of teaching and learning mathematics (see the topics in the 'mathematics' part of the list), while the distinction of the two human factors ('teacher' versus 'learner') is not too well taken into account. Only 'teacher education' differentiates the two roles in the struggle of human beings with mathematics. On the other hand, the categories of the 'environment' show that the didactical triangle (as a model) really forgets about important issues to be researched, while the two classifications on methodology (together with the 'epistemology' category) mark a certain meta-discussion on the scientific status of Didactics of Mathematics.

ICME-10, the 10th congress on Mathematics Education (for details on the series of ICME conferences, see the following section), gave additional information on international trends in Didactics of Mathematics. The "Survey Team 1" report entitled "What could be more practical than a good research? On mutual relations between research and practice of mathematics education" (see Sfard 2004), in its first descriptive part on research, stressed the fact that most of the research in Didactics of Mathematics has its "prevalent focus on the teacher and teacher practices … teacher-centeredness in research could be identified in ¾ of the respondents who claimed to be engaged in research." The concluding remarks of the report also give a historical sketch of the foci of research: "the last two decades of the 20th century … were almost exclusively the era of the learner … the era of the curriculum roughly corresponding to the 1960s and the 1970s", while "we may now be living in the era of the teacher as the almost uncontested focus of the researcher's attention". As for "research paradigms", the report sums up: "First, the basic type of empirical data is a carefully recorded classroom interaction … Secondly, this research emphasizes the broadly understood social context of learning … Third, the majority of the research is qualitative and does not make any reference to the quantitative argument".

Institutions: national and transnational

Research in Didactics of Mathematics is organised according to different patterns in different countries. In France, for instance, a research association (Association de Recherche en Didactique des Mathématiques – ARDM), organised on private initiative originating from the IREM movement, structures French research in Didactics of Mathematics through three different 'institutions', as follows. A peer-reviewed research journal (Recherches en Didactique des Mathématiques – RDM) is a forum for discussing research also internationally (with a slowly growing number of publications from outside France, especially from Italy and Spain). Three times per year, researchers meet in a 'national seminar' (at present starting Friday afternoon, lasting till Saturday noon). And as a meeting place every two years, a 'summer school' is held in different places. Three to four topics in Didactics of Mathematics are treated in depth by invited plenary lectures, seminars and working groups, accompanied by some less focussed activities such as presentations of ongoing research. These activities create a very strong identity among French researchers, not least because fluency in French is an informal but necessary condition for participating in the activities. It seems fair to say that three 'paradigms' clearly dominate the French speaking/publishing community of Didactics of Mathematics: the approach using 'fundamental situations' as starting points of research ('théorie des situations', originating from Guy Brousseau/Bordeaux; for a summarising English publication see Brousseau 1997), the 'anthropological approach' (with strong influence from its founder Yves Chevallard/Marseille), and the theory of conceptual fields (which has long been present, originating from earlier work of Gérard Vergnaud/Paris).

Didactics of Mathematics in **Germany** is less well organised, but the basic organisational unit is again a research association (Gesellschaft für Didaktik der Mathematik – GDM). GDM runs a peer-reviewed research journal (Journal für Didaktik der Mathematik – JMD) and a yearly conference in different universities (Tagung für Didaktik der Mathematik). The proceedings of these conferences (Beiträge zum Mathematikunterricht, published with the editor Franzbecker) give an easy overview of what is happening in Germany in terms of Didactics of Mathematics – and clearly show that the discipline is less structured and dominated by paradigms than the French research community.

In the **USA**, research in Didactics of Mathematics seems even less well organised than in Germany, although the PME group has an active North American subgroup (the Psychology of Mathematics Education, North American Chapter – PME-NA), which holds annual meetings[18]. The US teacher association (National Council of Mathematics Teachers – NCTM) edits the most prestigious research journal in the USA, the Journal for Research in Mathematics Education – JRME. A variety of journals appear in the US/Canada (e.g.: 'For the Learning of Mathematics – FLM'), some of them especially targeting the mathematics teacher (e.g. the NCTM-edited journal 'The Mathematics Teacher'). Special conferences are held throughout the year; the US scene is so rich and diverse that it is difficult to follow this from Europe.

Apart from the international associations already mentioned (ICMI, PME), there is a European initiative, 'Educational Research in Mathematics Education – ERME', which tries to further a European identity – not least through a series

[18] For more information see http://www.pmena.org/.

of conferences (Conference on Educational Research in Mathematics Education – CERME). In 1999 the first conference, CERME 1, was held in Osnabrück/ Germany. In 2001, CERME 2 was held in Mariánské Lázne/Czech Republic; in 2003, CERME 3 took place in Bellaria/Italy. CERME 4, the fourth in this series of biannual conferences, will be held in February 2005 near Barcelona/Spain. The CERME conferences differ from the majority of international conferences as they "deliberately and distinctively move(s) away from research presentations by individuals towards collaborative group work. Its main feature is a number of thematic groups whose members will work together in a common research area"[19]. For CERME 4 in 2005, 14 working groups are set up with the following topics: the role of metaphors and images in the learning and understanding of mathematics / affect and mathematical thinking / building structures in mathematical knowledge / argumentation and proof / stochastic thinking / algebraic thinking / geometrical thinking / mathematics and language / tools and technologies in mathematical didactics / mathematics education in multicultural settings / different theoretical perspectives / approaches in research in mathematics education / from a study of teaching practices to issues in teacher education / applications and modelling / advanced mathematical thinking (see the conference website again).

With the development of the field and with more specialisation taking place in the discipline, besides the general journals like "Educational Studies in Mathematics" and the "Journal of Research in Mathematics Education" more specific international journals emerged (e.g. the 'International Journal for Learning Mathematics with Computers – IJLMC', and the 'Journal for Mathematics Teacher Education – JMTE'). The International Commission on Mathematical Instruction (ICMI) also tries to motivate more focussed activities by organising 'ICMI studies'. These are international activities looking into special issues of international interest. They start with a 'discussion document' prepared by a committee and inviting comments and contributions to the issue to prepare an international conference where participation is only by invitation. After such a conference, a publication growing out of it is developed which basically builds on the presentations during the conference, but may also contain material from other sources. The topics of the last three studies were "The future of the teaching and learning of algebra" (ICMI study 12), "Mathematics education in different cultural traditions: A comparative study of East Asia and the West" (ICMI study 13) and "Applications and modelling in mathematics education" (ICMI study 14). The discussion document for ICMI study 15 on "The pro-

[19] From the website of CERME 4 at http://cerme4.crm.es/.

fessional education and development of teachers of Mathematics" has recently been published.

Finally, a rather recent and truly 'Nordic' development should be mentioned. In 2002, researchers from Sweden took the initiative to apply for a Nordic Graduate School in Didactics of Mathematics, when 'Nordisk Forskerutdanningsakademi – NORFA' offered a chance to apply. In the end, five of about 50 applications for graduate schools were retained by NORFA, one of them being a NORFA Graduate School for Didactics of Mathematics, hosted by Agder University College[20] and trying to develop a Nordic identity in Didactics of Mathematics.

Two complementary trends: case studies versus international comparison

If viewed in a very global way, two major methodological trends in international research in Didactics of Mathematics can be identified. In some sense, they are complementary, if not contradictory in that they are mutually exclusive as paradigms – even if research groups may be using both approaches. On the one hand, there are (for a majority) qualitative case studies, looking deeply into individual, limited cases. This approach can be characterised by attempting to cover "all" important features of a case (be it the solution process of a ten-year-old learner, the lesson preparations of a group of experienced teachers, or the development of a textbook series over time). The strengths of this approach are its openness to innovative ideas and the details of descriptions developed within it. Much research reported during PME conferences implicitly follows this approach – and according to the report at the ICME-10 conference, this is the dominant type of research in Didactics of Mathematics (see above in section 3.1; for details see Sfard 2004).

On the other hand, and getting ever more attention in recent years, there are large international studies which heavily rely on statistical comparisons – such as the Third International Mathematics and Science Study (TIMSS) or, in some contrast and competition with it, the OECD Programme for International Student Assessment, known as the 'PISA' study, again sponsored by OECD. In principle, the traditional statistical studies follow the same basic methodology, namely starting from a well-defined question (e.g. "Can a normal youngster in Western Europe aged 16 solve all types of quadratic equations?"), creating a set

[20] For more information see http://www.hia.no/realfag/didaktikk/forskerskolen/.

of (empirical) indicators to answer the question, testing an appropriate sample of the population to get information on the actual performance in relation to the indicators, and finally using statistical calculations and tests to decide on the initial question, often formulated as a hypothesis.

Both approaches obviously follow different paradigms – with TIMSS and PISA following a traditional statistical methodology, while the 'case study' approach implicitly favours an approach more 'grounded' in the reality of the educational enterprise (for a short description of 'grounded theory' as a scientific paradigm see Corbin & Strauss 1990; a description in Swedish is offered by Hartmann 2001). These two approaches have fundamentally different ways of coping with scientific generalisation. The case study approach – in a 'postmodern' mood – sometimes even denies the necessity or possibility of generalising beyond the individual case under study, whereas the traditional statistical approach was created for, and aims at, methodically controlling the generalisation from the cases under study to a wider population. In reality, all sorts of intermediate positions and research methodologies are used and reported in conferences and journals[21] – although some of the journals clearly favour one of these paradigms. To give two examples, it seems fair to say that the Journal of Research in Mathematics Education (JRME) has been favouring a traditional statistical approach, while For the Learning of Mathematics (FLM) is more or less devoted to the case study approach. There is another issue in the competition between case studies and international comparisons which should be explicitly mentioned: while international comparisons can only be done if large sums of research money are found and spent, case studies are within the reach of an individual researcher using locally available funding. As a consequence, the battle between those two paradigms is not only a methodological or epistemological one; it heavily depends on the economic resources and limitations of the researcher or research group.

[21] In this respect I disagree with the statement of Sfard 2004: "The gulf that separates the qualitative and quantitatively inclined math ed researchers appears difficult to bridge." I start from the assumption that in this respect the sample in the survey team report was somewhat distorted. The "Research Forum 04" on "Contrasting comparative research on teaching and learning in mathematics" of the PME 28 conference in 2004 (jointly organised by a Swedish and an Australian researcher), and the fact that this Research Forum attracted a wide audience, can be taken as an indication of a narrowing gap between quantitative, comparative, large- scale studies and qualitative, small studies – if this gap exists at all.

Swedish and international research

Swedish researchers publishing internationally

An easy way to judge the intensity with which Swedish research in Didactics of Mathematics is presented internationally is a count of the number of Swedish authors in PME proceedings (for the PME group, its conferences and its relevance for research in Didactics of Mathematics, see section on national and transnational institutions). This count ends with a real deception: during the three years 2000, 2001 and 2002, no Swedish author is mentioned in the list of authors of the respective PME proceedings! For 2003, one Swedish author is listed in the proceedings of the PME conference in Honolulu/Hawaii. For 2004, the proceedings of the PME conference in Bergen/Norway list five Swedish authors – I myself, Rudolf Strässer, was classified as German, even though I am known to be a professor in Luleå/Sweden. In addition to this, the Swedish KULT project was presented as a major part of the international "Learner's Perspective Study" (for more information on this international enterprise see below). In all the five conferences from the year 2000, smaller countries like Finland or Norway had more authors listed as authors in the PME proceedings. It seems to follow that Swedish Didactics of Mathematics is not very well presented in the most important international conference on research in Didactics of Mathematics.

This result is confirmed by another quest for the presentation of Swedish research in an international arena. Checking the last ten years of the two most important scientific journals in Didactics of Mathematics ('Educational Studies in Mathematics – ESM' and 'Journal for Research in Mathematics Education – JRME'), one comes across only two publications with Swedish authorship, namely by one author who published in ESM (see Lithner 2000 and 2003). This is a clear confirmation of the fact that Swedish research in Didactics of Mathematics did not get off the ground until the late 1990s. The total of two Swedish papers published in these journals illustrates very well that an international visibility is only slowly developing in the 21st century.

Experience of the recent International Congress on Mathematics Education ICME-10 in Copenhagen shows that the Swedish researchers really make an effort to change this situation. One of the 79 'regular lectures' was given by a Swedish researcher (Christer Bergsten on "Exploiting the gap between intuitive and formal knowledge in mathematics"), and two Swedish researchers (Lithner and Strässer) acted as chief organisers of two of the 29 'Topic Study Groups'; no Swedish colleague was involved in the management of the 24 'Discussion Groups' at ICME-10. It seems impossible to ascertain the number of individual presentations given by Swedish researchers during the whole congress. At least in the ICMI environment, the Swedish research community makes a distinct effort to be visible internationally.

Finally I want to mention an international project with an important Swedish participation, the *"Learner's Perspective Study"* (LPS). In this comparative study, researchers from Australia, the Czech Republic, Germany, Hong Kong, Israel, Japan, the Philippines, South Africa, Sweden and the USA try to avoid the divide between quantitative, large-scale and qualitative, small studies. Its Swedish section, *KULT-project*, is run by persons from the Department of Education at Göteborg University (team leader in Göteborg: F. Marton) and investigates mathematics teaching in lower secondary schools, more specifically the teaching in higher grades of the grundskolan[22]. Typically, at least for the Göteborg situation, the Swedish section is run by pedagogues, by institutions 'outside' Swedish Didactics of Mathematics – even if it is one of the larger comparative studies internationally. At the moment, the project seems to follow the gradual shift from the national "lesson script" (the first TIMSS video-study with Germany, Japan and the US participating in 1995) over a more detailed, still mainly quantitative analysis of individual lessons (the 'TIMSS video-repeat study' in 1999, with Australia, the Czech Republic, Hong Kong, South African Republic, the Netherlands, Switzerland and the United States participating[23]), to the analysis of "lesson events" in the Learner's Perspective Study. From the little material publicly available especially on the KULT project, one gets the impression that the LPS as a whole somehow tries to bridge the divide between quantitative and qualitative studies in Didactics of Mathematics.

Concluding remarks – suggestions

Following from the above description of Swedish and international research in Didactics of Mathematics, a global characterisation of the position of Swedish Didactics of Mathematics seems reasonable: while begun rather recently in the 1990s, Swedish research is well developed in some fields. Research in the

[22] For details see http://www.ped.uu.se/kult/default.asp.

[23] See http://www.lessonlab.com/timss1999/.

paradigm of phenomenography (with interviews as the major methodology) is definitely a strength and specific feature of Swedish research in Didactics of Mathematics. The 'Prim' group at the Teachers' College in Stockholm (for primary and lower secondary education) as well as the mathematics group at the Department of Educational Measurement in Umeå (for upper secondary mathematics education) represent a research tradition in design and evaluation of national tests, which Sweden should offer to the international research community. Research on textbooks (recently begun in Luleå) is well in line with the graduate school 'DidaktikDesign' at Lärarhögskolan Stockholm[24] and may be another Swedish research area of international interest. According to its website, the 'DidaktikDesign' research school plans a *Centrum för utvärdering och bedömning* with the support of Astrid Pettersson, which could be a hint of future research linked to mathematics and linking the two topics of assessment and textbook research. This is also in line with international activities like the 'IARTEM' network (International Association for Research on Textbooks and Educational Media; its president is Staffan Selander from Lärarhögskolan Stockholm).

If compared to the international situation, Swedish research in Didactics of Mathematics is still lagging behind in terms of organisation and 'depth and breadth' of research. One should also mention that Swedish research has not (yet) taken into account the recent swing from learner studies into teacher-centred studies. The "International Symposium on the Preparation of Mathematics Teachers for the Future" (for the proceedings see Strässer et al. 2004) held in Malmö in May 2003 can be seen as an effort to overcome this deficit. The deficit is particularly astonishing since research-based teacher education was made a national topic of concern by the Swedish government, not least with its latest reform of Swedish teacher education some years ago. In this respect, the Swedish research community is following neither the international trend nor the needs identified by the Swedish government.

Judging from the international experience, there are two main ways to promote the fast development of Swedish Didactics of Mathematics. Firstly, building some local centres can be a suitable catalyst. The continuation of graduate schools like the RJ/VR would be an excellent means of creating such local centres. From my personal experience within Germany, I want to opt explicitly for a plurality of such centres – not a single national central agency. Plurality of centres gives rise to competition and avoids the creation of 'elites' defined only by affiliation to a certain institution. With the excellent infrastructure given

24 See www.didaktikdesign.nu.

in Sweden, this multi-centre approach can be backed by a national reference library and documentation centre (like the one already under development at NCM in Göteborg) – but it should explicitly aim at creating more than one or two 'centres of excellence' in Didactics of Mathematics. Creating national 'centres of excellence' will automatically imply two consequences, which would additionally support the national, Swedish development: a more visible presence of Swedish researchers in the international, non-Swedish research community and hopefully more Swedish publications in international journals.

Secondly, Sweden can and should profit from the existence of a well-developed international research community in Didactics of Mathematics. The obvious way to do so is a continuous practice of inviting researchers from outside Sweden to lectures, seminars and conferences – if not offering them temporary positions as guests in Swedish institutions under development. The practice of the RJ/VR graduate school – inviting foreign lecturers to give the courses for the graduate students – is an example worth imitating. Given the few persons fully qualified at professorial level in Sweden, Swedish institutions interested in developing Didactics of Mathematics as a research domain should try international recruitment (even if difficult) to speed up the development of Didactics of Mathematics.

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Strässer, R., Brandell, G., Grevholm, B., & Helenius, O. (Eds.) (2004). *Education for the Future. Proceedings of an International Symposium on Mathematics Teacher Education*. Göteborg / Stockholm: NCM / The Royal Swedish Academy of Sciences.

Wistedt, I. (2004). *Forskning om lärande i matematik, naturvetenskap och teknik*. From http://www.ped.su/uploads/files/597forskning_wistedt.pdf. (downloaded June 28, 2004).

Appendix A

Anna Friberg and Rudolf Strässer

List of Swedish dissertations and licentiates in Didactics of Mathematics

This appendix presents a list of dissertations and licentiates, including abstracts where available, in Didactics of Mathematics (mathematics education) which have been finished in Sweden or by Swedish researchers. First, the Ph.D. theses are listed, followed by a list of the licentiates. Within these two categories, the titles are ordered according to year of publication, starting with the oldest and moving forward in time. Information about titles and other interesting facts have mainly been collected from http://ncm.gu.se/index.php?name=forskning-litteratur-doktorsavhandlingar.

The list of licentiates includes only the ones found on the website http://ncm.gu.se/index.php?name=forskning-litteratur-licavhandlingar. To the best of our knowledge, it is nevertheless as complete as possible.

1 List of Ph.D. Theses

- **Johnsson, K. G.** (1919). Undersökningar rörande problemräkningens förutsättningar och förlopp. [Investigations on the preconditions and process of problem calculation.] Uppsala: Almqvist & Wiksell.

Abstract *(translated from Swedish by Swedish Research Council)*
These investigations have shown clearly that great differences exist in regard to problem-solving. The factors influencing it can be of both relatively external and internal kinds. Differences in the formulation of problems may have the result that solutions of one and the same task differ entirely in character in purely mathematical terms.

We have also found that the problems' own nature has a strong influence. For example, a formula problem does not receive the same treatment as a so-called general task.

The problem treatment may acquire its special character from the use of a

certain kind of calculation. A calculation employing equations is often different than when the problem is solved in another way.

Also influential is the manner of learning. Both teachers and textbook authors may have peculiarities whose influence is detectable.

The state of the individual who is calculating can be a source of differences in the manner of working. Indisposition of one sort or another usually lowers performance and gives it a more primitive form.

The individual's endowment of knowledge, talent and interest in relation to the assigned problems may impart a special character to the work of analysis and synthesis. A relatively well-endowed individual does not, as a rule, solve a problem in the same way as a worse-endowed one. The farther the individual has progressed in the subject, the more accomplished his manner of working becomes. Our experimental persons have been seen to pass over from associative into logical, and then towards intuitive, analysis and synthesis.

However, the persons taking part in these investigations also exhibited some differences concerning problem treatment that could not have depended on the above circumstances. For instance, one person made persistent use of exhaustive illustration, even when this should not have occurred in view of the causes that were enumerated. Another person rather consistently tended to oppose such a procedure. Corresponding contrasts occurred in connection with fundamental and non-fundamental analysis, as well as with the unconscious components.

Differences in illustration, method of analysis, and unconscious components are nonetheless plausibly assignable to a common origin: differences in analysing and synthesising. Here we distinguish between what are called associative, logical, and intuitive analysis and synthesis. The associative approach is often characterised by an attack on the problem at random, the logical by a detailed dissection in a certain order, and the intuitive by a comprehensive perspective.

The process of problem-solving can be divided into three principal phases: the preliminary, main, and follow-up periods. Between them, however, the boundaries are flexible. The preliminary period, which lasts approximately until one is acquainted with the problem and begins to analyse it, is characterised by waiting for what comes next – an initial suspension and orientation. The main period, during which the actual work of solution is performed, is characterised by analysing and synthesising, which may differ according to the person calculating. The follow-up period is characterised by evidence and "feeling for correctness".

Components of emotion and volition occur in problem-solving to a greater extent than one may believe. Still, large individual differences exist even in that respect.

- **Wictorin, M.** (1952). *Bidrag till räknefärdighetens psykologi: en tvilling-undersökning. [Contribution to the psychology of skill in calculation: a study of twins.]* (Akad. avh. Göteborgs högsk.) Göteborg: Gumpert.
  University of Gothenburg.

No abstract available.

- **Werdelin, I.** (1958). *The mathematical ability: experimental and factorial studies.*(Studia psychologica et paedagogica. Series altera, 9.) Lund: Lund University.Lund University.

No abstract available.

- **Dahllöf, U.** (1960). *1957 års skolberedning. Kursplaneundersökningar i matematik och modersmålet – empiriska studier över kursinnehållet i den grundläggande skolan. [1957 School Commission. Curriculum investigations in mathematics and Swedish – empirical studies of course content in the basic school.]* (Statens offentliga utredningar, 1960:15.) Stockholm: Stockholms Högskola.
  Stockholm University.

No abstract available.

- **Postlethwaite, N.** (1967). *School organization and student achievement:a study based on achievement in mathematics in twelve countries.* (Stockholm Studies in Educational Psychology, 0562-1089: 15.)
  Stockholm University.

No abstract available.

- **Ekman, B.** (1968). Teaching geometry in grade four: three experimental studies carried out in a practical educational situation.
  (Studia scientiae paedagogicae Upsaliensia, 10.) Stockholm.

No abstract available.

- **Larsson, I.** (1973). *Individualized mathematics teaching: results from the IMU project in Sweden*. (Studia psychologica et paedagogica. Series altera, 21.) Lund: Lund University.
  Lund University.

No abstract available.

- **Holmberg, I.** (1975). *Effects of some trials to improve mathematics teaching.* (Studia psychologica et paedagogica. Series altera, 26.) Lund: Gleerup.
  Lund University.

Abstract

1. In the *achievement variable mathematics,* the different methods produced equivalent results in both experiments. In the standard tests for grade 8 there was in the spring term of '64 no significant difference between any of the experiment classes and the national population. On the other hand the students of the machine class with a maximum use of teaching machines were significantly better than those of the comparison class.
2. In *reading ability* no differences occurred between the classes compared if we consider the experiment period as a whole. According to the standard tests and standardized reading tests the experiment groups are representative of upper level students in general. Reading comprehension increases steadily for students with a maximum use of teaching machines, while reading speed fluctuates from occasion to occasion.
3. *Behavior:* More disturbances occurred in classes with conventional teaching than in classes with machine teaching combined with group teaching.
   At the end of each term the disturbing activities increased in classes with conventional teaching.
   Behavioral differences between high, middle and low groups were only noted in experiment 1 and then in the variable "Disturbing interactions." The middle group here had higher scores than the others, especially towards the end of the experiment period in grade 8. Disturbances encroached on very little of the lesson time, however. The great variation between the test occasions for all behavior variables depends on irregularities in the time-pattern, rather than on a distinct trend.
4. *Preference for mathematics,* measured by Form H, remained relatively constant for the comparison classes. In the class with maximum use of teaching machines the preference diminished, but there was still no significant difference between the machine and comparison classes. In the class with the modified experiment (group teaching + machine teaching) the preference values for the whole experiment period lay somewhat above those for the comparison class, but no difference between the methods could be established.
5. *Attitude towards machine teaching* has been studied by means of essay analysis in Experiment 1. The reaction of the students in class 7 was fairly positive. This changed in grade 8. The students were significantly more negative in class 8 than in class 7. After three terms' teaching by machine 15 out of 16 students had a negative attitude towards P1. This result should not be overinterpreted. We have no information as to how negative the comparison class was towards its mathematics teaching in grade 8.

In Experiment 2 the attitudes towards machine teaching were evaluated by means of Attitude test H. When the scores on all the statements in the test were added and the difference in attitude between grade 7 and grade 8 was tested, a negative bias was noted in this modified experiment too. This difference stemmed mostly from the circumstance that the students considered programmed teaching more tiring than conventional teaching. After three months' work with machines, 73 % of the students found this form of work more tiring. 67 % of the students thought that they learned more mathematics or as much as they would have done with conventional teaching.

Thus in our two long-term experiments – the first with maximum use of teaching machines and programmed material and the other with programmed material presented via machines alternating with teacher-led small-group teaching, both compared to conventional teaching – no significant differences were found in mathematics achievement or in preferences for the subject. Behavior during mathematics lessons is rather similar for students using the three different methods. The attitudes towards the method with maximum use of teaching machines are negative. When we modify the method by alternating group-teaching and machine-teaching, the attitudes become more moderate. The students consider that programmed teaching is more tiring than conventional teaching, but in compensation they think they learn more. This opinion is to a certain extent verified by the results obtained on the standard tests, which were given three weeks after the completion of Investigation of Effects 1.

In the standard tests in mathematics, spring term '64, the class with a maximal use of teaching machines was significantly better than the comparison class. This is an important result, since the standard tests consist of centrally produced test material. The good results achieved by the machine class in the standard tests show that the programmed material has given the students good training in various mathematical skills. The doubts expressed earlier that PI might make the students less able to solve problems were not confirmed. The students in the machine class achieved these good results in the standard tests despite the fact that they have had less time for mathematics training than those in the comparison class, who had homework once a week.

- **Noonan, R. D.** (1976). *School resources, social class, and student achievement: a comparative study of school resource allocation and the social distribution of mathematics achievement in ten countries.* (IEA monograph studies, 5.) Stockholm: Almqvist & Wiksell International. Stockholm University.

Abstract
An empirical study of the allocation of resources among schools in ten countries was carried out. The ten countries included England, the Federal Republic of Germany, Finland, France, Israel, Japan, the Netherlands, Scotland, Sweden, and the United States. Allocation of resources was examined in connection with the socioeconomic status and level of achievement of students served by the schools. An attempt was made to assess the instrumental value of the school system in reducing achievement differences associated with socioeconomic status.

In each country the sample of schools was partitioned into groups on the basis of socioeconomic status variables. Groups of schools were compared within countries to determine the extent to which resource allocation varied. On the basis of these comparisons and multivariate analyses based on the total sample of schools, countries were classified as having either Elitist or Egalitarian Resource Allocation. A cross-national comparative analysis was then used to explore the impact that Mode of Resource Allocation and School System Structure had on the level and social distribution of student achievement.

No evidence was found of an effect of Mode of Resource Allocation or School System Structure on the level and social distribution of student achievement. It was therefore suggested that societal factors have the dominating influence on both the school system and school outcomes.

Some analysis problems and policy implications are discussed.

- **Håstad, M.** (1978). *Matematikutbildningen från grundskola till teknisk högskola i går – idag – i morgon. [Training in mathematics from grade school to technical university yesterday – today – tomorrow.]* (Trita-EDU, 016.) Stockholm. University of Uppsala.

Abstract
A didactical analysis of some aspects of mathematics learning and teaching is made including the separation of the goals into sub-goals, distinction between degrees of knowledge as to the way mathematics is used, and the concept of potential of a mathematics education. A study is made of the development during the last decade of the mathematics teaching in Sweden including the introduction and decline of "New Mathematics", the consequences of the far-reaching restructuring of the school system and an analysis of which persons have the greatest influence on the development. The extent to which mathematics teaching has been reduced and the classes made more heterogeneous is measured. The mathematics teaching in higher technical education is analyzed,

based on interviews and questionnaires with students, teachers and examined engineers. Suggestions are given for future teaching.

- **Kristiansson, M.** (1979). Matematikkunskaper Lgr 62, Lgr 69/Knowledge of mathematics curriculum 62, curriculum 69. (Göteborg Studies in Educational Sciences, 29.) Göteborg: Acta Universitatis Gothoburgensis. University of Gothenburg, Department of Educational Research.

Abstract

The background of this investigation is the changes in teaching of mathematics which were made in connection with the revision of the 1961 curriculum of the nine-year compulsory comprehensive school in Sweden. Through this revision the new math was introduced in the new curriculum of 1969.

The main aim of this investigation was to compare the results of teaching mathematics according to curriculum 62 (old math) with those of curriculum 69 (new math). This investigation was carried out in form 3 and form 6. The selection of pupils was made within three different strata. Stratum 1 was a city, stratum 2 places with comprehensive schools and senior higher schools and stratum 3 places with only comprehensive schools. The comparisons were made both between the total E(62) and the total E(69) and between E(62) and E(69) within each stratum. Comparisons between different strata within E(62) and within E(69) were also made.

Three tests containing items of mental arithmetic, rough calculation, arithmetic and problem solving were used in this investigation. In addition there were two questionnaires, one to the pupils about interest in mathematics and one to the teachers.

The main results were: In the comparisons between E(62) and E(69) pupils in form 3 have about the same result. But in form 6 the total result shows significant differences to the advantage of pupils in 6(62). The results of comparisons of different types of items show that E(62) in both forms is superior to E(69) in arithmetic. But there are no differences in problem solving.

The comparisons between each stratum vary somewhat. In stratum 3 "countryside", there are no significant differences at all. In the comparisons between different strata within E(62) and within E(69) there were about as great variations in E(62) as in E(69).

Independent of any curriculum, mathematics is a popular subject in both forms.

The varying results of this investigation point out that the deciding factor for the outcome of the teaching of mathematics is not the content of the cur-

riculum as such but depends to a high degree on how the teaching is carried on. What is more important for the future teaching of mathematics is to discus why we got these results and what can be done to improve the teaching. The dissertation concludes with discussion of problems in the teaching of mathematics.

- **Allwood, C. M.** (1982). *Knowledge, technique, and detection of errors in statistical problem solving.* (Göteborg Psychological Reports, 12:2.) Göteborg: Göteborgs Universitet.
  University of Gothenburg.

Abstract
This thesis consists of five papers one of which summarizes the other four. The thesis deals with novice problem solvers' use of knowledge and detection of errors. In addition, how solvers generate errors, and differences between good and poor solvers, are investigated. Problem-solving is seen as a process, the outcome of which depends on the solvers' knowledge and how that knowledge is utilized (problem solving technique). In three of the studies, the subjects were asked to think aloud while solving two (or three) statistical problems. The empirical results of the studies include the following. Solvers' use of knowledge was found to be deficient in various ways, e.g. the use of conceptual knowledge was found to be insufficient. The ability to detect one's own errors was found to be important for successful problem solving. Different aspects of the solvers' detection of their own errors were studied. Most of the error detections were found to occur either in a quick one-step process or in a prolonged multi-step process triggered when the subjects reacted in a negative way to some aspect of their solution. Various methods to make subjects detect their own errors were attempted and the causes of the moderate success of these attempts were analyzed. Finally, good and poor solvers were, generally speaking, found to differ in the degree to which they utilized an understanding-oriented approach. Good solvers' greater ability to detect their own errors appears to depend partly on advantages in the early stages of the error detection process. Some practical implications of these and other results are discussed.

- **Warg, L-E.** (1983). The effect of task content on performance in probabilistic inference tasks. (Abstracts of Uppsala dissertations from the Faculty of Social Sciences, 28.) Uppsala: Uppsala Universitet.
  University of Uppsala, Department of Psychology.

Abstract
The purpose of the present thesis was to investigate the effect of task content on performance in probabilistic inference tasks which involve uncertainty. A series of empirical studies were conducted within the general framework of Social Judgment Theory (SJT), which stems from the work of Egon Brunswik. Before the three empirical studies are summarized, the framework is briefly described, and the possible effects of task content are discussed as well as the results of earlier studies concerned with effects of task content on performance in inference tasks.

The results from the empirical studies showed that (i) meaningful task content has a facilitating effect on performance in probabilistic inference tasks and that on a formal level of description, the effect is due to the fact that content provides information about the function forms of the cue-criterion relations and the relative weights of cues, (ii) subjects tend to follow a strategy suggested by content, at least so long as the outcome feedback values do not deviate too much from the expected values, (iii) subjects can perform well in tasks with incongruency between content and actual task structure by redefining the task so that they can make use of their old knowledge, (iv) the knowledge applied by subjects in content tasks can be causally organized, and (v) effects of predictability and function forms obtained in studies employing abstract tasks are valid also in tasks with content.

Finally, it is suggested that some research efforts in the future should be directed at the following topics: investigating the frequency of causal organization of knowledge, exploring the limits for the effect of task content pointed to in the present thesis, and finally, trying to find out how subjects choose combination rules in different content tasks.

- **Hellström, L.** (1985). *Undervisningsmetodisk förändring i matematik: villkor och möjligheter. [Changes of instruction method in mathematics: conditions and possibilities.]* (Studia psychologica et paedagogica. Series altera, 79.) Malmö: Liber Förlag/Gleerup.
  Lund University, Department of Educational Research, Malmö.

Abstract
This dissertation aims to cast some light on the problem of change in mathematics instruction in the compulsory school (“grundskolan”). In spite of centrally initiated steps towards changes of mathematics instruction in connection with the introduction of a new curriculum in Sweden (“Läroplan for grundskolan, 1969”), there are indications which seem to support the conception that changes have been limited or non-existing.

Within the framework of a case study, the aim of which was to initiate a change of mathematics instruction and describe and to some extent explain critical phases of the change process, a study of change has been carried out by the flexible use of different kinds of information.

The scientific approach could be described as Action Research where observations, questionnaires and interviews have been used to describe teachers' reactions and conceptions. Participating pupils have been tested by the use of Raven's matrices and an attitude scale. A sample of pupils have also been interviewed as well as the headmasters in the municipality concerned.

In discussing the results different theoretical perspectives have been used: organizational, ecological and individual.

The results indicate that a practice-centered in-service training could promote changes in mathematics instruction. A major conclusion is that change has to be considered a problem that concerns both the teacher and the educational setting, meaning a development of individual competence as well as developing the school as an organization. Some consequences for future in-service training are presented.

- **Neuman, D.** (1987). *The origin of arithmetic skills: a phenomenographic approach.* (Göteborg Studies in Educational sciences, 62.) Göteborg: Acta Universitatis Gothoburgensis.
  University of Gothenburg.

Abstract

"If teachers knew about, and used, children's own ways of learning maths, the teaching of maths by adults would end up in the learning of maths by children". This polemic was the hypothesis which the present study took as its point of departure.

As an introduction, two apparently conflicting phenomena are discussed. On the one hand pupils who experience difficulties with mathematics, even at the end of their school life demonstrate that they lack the most basic concepts of the ten numbers which constitute the arithmetical system used throughout the Westernized world. At the same time, experiences with some 25 different first-grade classes suggest that in any group of a score of children about to start school (7-year-olds in Sweden), one or more will seem to have already developed these concepts.

The investigation consists of two parts: an interview study with school starters, and a teaching experiment. The purpose of the interview study, which was carried out before any formal teaching started in the years 1982 and 1983, was

to answer the question: "what are the informal ways in which preschool children form concepts of the ten basic numbers?" To answer this question 105 school starters were given simple word problems, and were asked to describe the strategies they used to solve them. Their answers were tape-recorded and transcribed word-by-word.

The study is an example of phenomenographic didactics. The children's own strategies, and the conceptions these strategies were thought to express, formed categories of description. These were used as a guide in the teaching experiment, in which 39 of the children who were interviewed in 1982 took part, and in which the other children interviewed in the same year formed a control group.

Twenty-four children had demonstrated that at the time they started school they had neither established the pre-concepts, nor acquired the counting skills, which they would need to be able to follow traditional maths teaching in grade 1. These children, 11 in the experimental group and 13 in the control group, were re-interviewed two years later. The results of these interviews showed that while at least nine of the thirteen children in the control classes had still not formed the concepts of the ten basic numbers, all the children in the experimental classes had done so and could also apply this understanding in more complex problem-solving.

The results of this investigation tend to confirm the starting hypothesis:

- that, even in their preschool days, children develop without formal instruction the concepts they need to solve their own everyday problems, adequately and efficiently;
- and that if teachers use the children's own concepts as the starting-point for more advanced thinking, then the teaching of mathematics will assuredly result in the learning of mathematics.

- **Bergsten, C.** (1990). *Matematisk operativitet: en analys av relationen mellan form och innehåll i skolmatematik. [Mathematical operativity: an analysis of the relation between form and content in school mathematics.]* (Linköping Studies in Education Dissertations, 29.) Linköping: Linköping University. Linköping University, Department of Education and Psychology.

Abstract

In this thesis interest is focused on the relationship between conceptual and symbolic-notational aspects of elementary school mathematics. The literature indicated that students' conceptions of the integration of content and form are essential for their understanding of mathematics.

A unit of analysis, the "crystal", is based on the distinction between content and form of mathematics, on the one hand, and between structure and operation, on the other. The interplay among these four dimensions of mathematics is discussed with respect to its logical and psychological aspects.

The notion of mathematical form, modelled by formal systems, is defined as the spatial relationships between typographical units in written symbolic expressions. It is found that empirically based mathematical content is reflected in "genetic" mathematical forms. Mathematical operativity is then identified as the ability to integrate, in mathematical problem solving, the four dimensions of the "crystal".

An empirical study, including written tests and interviews, involving 54 students in grades 8 and 9 of the Swedish compulsory school, showed that conceptual and formal understanding are equally related to performance in school mathematics, while logical understanding could be viewed as a mediator for the creation of conceptual-formal links.

Finally, implications of the study for the teaching of elementary school mathematics are discussed.

- **Hedrén, R.** (1990). *Logoprogrammering på mellanstadiet: en studie av fördelar och nackdelar med användning av Logo i matematik-undervisningen under årskurserna 5 och 6 i grundskolan. [Logo programming at the intermediate level: a study of advantages and drawbacks with using Logo in mathematics instruction during forms 5 and 6 in the primary school.] (Linköping Studies in Education Dissertations, 28.)* Linköping: Linköping University. Linköping University, Linköping Studies in Education.

Abstract

This thesis deals with a teaching experiment on the teaching of mathematics with the help of the programming language, Logo, in forms 5 and 6 (age groups 11–12) in the Swedish primary school.

The thesis starts with a constructivistic theory of learning, especially applied to mathematics, as it has, above all, been rendered by Richard Skemp. After that, some theories and research results on pupils' comprehension of numbers and of geometric concepts are discussed.

A rather exhaustive part of the thesis deals with the programming language Logo. The construction of the language and its potential advantages for pupils' learning of Logo. The construction of the language and its potential advantages for pupils' learning of mathematics are discussed, as are the thoughts and intentions of Seymour Papert, the originator of the language, when introducing this language as an aid to children's thinking. The results that have been achieved

when using Logo in various ways in research projects in, above all, the USA and the United Kingdom are discussed. The language and its use in school instruction have been criticized by various authors, and this criticism is also considered.

The aim of the project has been to investigate – with the theory accounted for as a background – whether pupils in the Swedish primary school may profit from programming in Logo during one period per week, especially whether this occupation may affect their knowledge of arithmetic and geometry and their problem solving ability. The aim has been summarized in the following problem specification:

How are pupils in forms 5 and 6 of the Swedish primary school affected when given the opportunity to program in Logo during one period per week in accordance with a work plan, in which there are great possibilities for the pupils to investigate and to experiment on their own:

1. Is there a change in the pupils' knowledge of geometry and arithmetic?
   Is there a change in the pupils' conception of number?
   Is there a change in the pupils' ability to solve problems?
2. How much do the pupils grasp of the ideas behind Logo programming?
3. Is the pupils' attitude to computers and the information society affected, if they are given the opportunity to program computers at junior school?
4. Is the pupils' attitude to mathematics as a subject affected?
   Is there any difference here between boys and girls?

In that case can this difference affect the development of knowledge in math ematics of boys and girls in different ways?

The experiment was carried out in two classes that were followed through forms 5 and 6 during the school years 1986/87 and 1987/88. Two comparison classes from the same school management area and with as equal prerequisites as the experimental classes as possible were also part of the project. Work sheets, which had been written by the author, with about ten sections per school year were used. In each such section a new programming technique or in some cases a new mathematical idea was presented to the pupils. Every section was built up in the following way:

The pupils got a new instrument to work with. Typical examples were given. The pupils worked on their own with given figures and similar exercises. The pupils invented their own figures and other exercises.

Tests in mathematics and programming, clinical interviews on mathematics and programming, questionnaires, interviews with pupils and with teachers, and observations were used as methods of evaluation. The results of this evaluation are exhaustively accounted for in the thesis. In addition there are case studies from pupils with different pre-knowledge in mathematics.

In the thesis there are numerous examples of how the pupils of the experimental classes were able to take advantage of the way they worked with discovery and investigation in Logo. It can also be seen how an erroneous step – seen from the point of view of an intended result – could still lead to an interesting and very often amusing figure. In that way the pupils got an attitude to mathematics that differed from that usual in traditional mathematics instruction; they discovered that a problem might be solved in different ways, that you can examine different ways to tackle a problem in a creative way, and that there is not necessarily one correct and one false answer to every question in mathematics.

The teacher's role was also different in Logo instruction compared to traditional instruction. Her/his task was more to organize the work of the class, to encourage the pupils to reflect on what they are doing, and to give them work tools, ideas, and suggestions, when they feel a real need for them. The pupils also had the opportunity of working together and in that way to compare and discuss each other's trains of thought.

In the way mentioned above the pupils had numerous opportunities to learn and to work with concepts and concept structures. In the thesis there are many examples of how the pupils assimilated their experiences at the computer in their ready-made schemas, and of how their schemas were differentiated, expanded, and also reconstructed. The pupils' motivation for computer work was strong, even if some pupils got less enthusiastic towards the end of the project time.

The drawbacks that instruction with Logo might give rise to are also discussed in the thesis. It sometimes happened that the system and the application programs caused trouble, and that the teacher had difficulties in finding the error. Above all the technical details, which you always need to know to be able to use a programming language, might cause the pupils unnecessary trouble. The formal side of Logo might hide the mathematical content that the instruction with Logo was intended to make clear to the pupils. Moreover the Logo environment tended to be a special subjective domain of experience to the pupils, which had very little in common with school mathematics or with the informal mathematics that they used outside school.

The teacher in one of the experimental classes, f 1, used Logo more freely and openly than the other experimental teacher, i. e. he let his pupils work more on their own and restricted his own teaching to a very minimum. This fact has been supported partly by interviews with the teachers and partly by tape recordings from Logo periods, one from each class. The difference in teaching style was mirrored in the test results. The quantitative evaluation showed that the result development was better in the experimental than in the comparison classes in arithmetic, in geometry, and in problem solving. However, the latter result was not significant, and is thus uncertain. The results in arithmetic and geometry were wholly due to the development in class f 1. The motivation for mathematics also developed more positively in the experimental than in the comparison classes and again this was above all the fact in class f 1. The central results are summarized in figure 1.

On the **upper** line the branches of mathematics where the comparison classes showed better results have been reported to the left and those where the experimental classes had better results to the right. On the next line the results from the two experimental classes are compared in the same way, and on the third line the results from girls and boys respectively in the experimental classes are seen.

The benefits of Logo were, thus, that it gave the pupils the possibility, even forced the pupils, to employ an active, exploring, and discovering style of learning. The pupils' own ideas and suggestions were carried out, and the computer gave an immediate and automatic feedback. But at the same time one or more erroneous commands would still lead to an interesting and often amusing result.

- **Pettersson, A.** (1990). *Att utvecklas i matematik. En studie av elever med olika prestationsutveckling. [Being developed in mathematics. A study of pupils with different performance development.]* Lärarhögskolan Stockholm, Stockholm. Stockholm University, Department of Education.

Abstract

The main purpose of this study is to analyse how pupils, differing in achievement levels in mathematics, solve mathematical problems. What strategies are used when they solve a problem correctly and what types of errors are involved when they solve a problem incorrectly? Groups of pupils who have shown different developments in mathematical achievement from grades 3 to 6 have been studied.

The pupils' solutions have been classified as correct, incorrect or not answered. The strategies of the solutions are analysed and if the solution is wrong the error is classified in one of two categories, grave error or simple error. Grave errors are, for example, when the pupils use incorrect rules of arithmetic, "borrow a ten" incorrectly or use 1 hour = 100 minutes. Simple errors are errors in the calculations and slips.

The pupils very seldom used mental arithmetic. However, they did so more often in grade 3 than in grade 6. When they solved a problem they frequently used a standard model. When the pupils solved problems correctly they used the same strategies irrespective of their total results. However, when they made errors they used a number of different strategies.

When the pupils who reached good results in both grades made a mistake, it was almost always a simple error. The pupils who had poor results in both grades made a great number of grave errors. The grave errors continued in grade 6. This group of pupils left the problems unanswered to a greater extent than the pupils in the other groups. One group of pupils have vastly improved their results to grade 6. In grade 3, however, they made grave errors to almost the same extent as the group with poor results in both grades. In grade 6 this group made mostly simple errors. One group of pupils performed poorly up until grade 6. This group made a larger number of grave errors in grade 6 than in grade 3.

When we become aware of the strategies the pupils use to solve a problem incorrectly we can, once again, establish the fact that their previous knowledge and skills are very different and, consequently, it is most essential that the teaching be more individualised.

- **Ahlberg, A.** (1992). *Att möta matematiska problem: en belysning av barns lärande. [Meeting mathematical problems: an elucidation of children's learning.]* (Göteborg Studies in Educational Sciences, 0436-1121; 87.) Göteborg: Acta Universitatis Gothoburgensis.
  University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract
The aim of the empirical study is to describe and analyse how primary school children in a school context experience arithmetic word problems and problem solving. The investigation has been carried out with a phenomenographic research approach and includes both an interview and a classroom study. In the interview study deep semi-structured interviews were held with 38 primary school pupils while they worked at solving problems from a sequence consisting of five different types of problem. In the classroom study three school classes were studied twice a week for a term while they worked with the same problems as the pupils in the interview study. In the classroom study problem solving was studied with the point of departure that pupils should draw, write, talk and calculate while solving problems. The classroom has a quasi-experimental design with pre- and post-tests in the participating and control classes, since one partial aim of the study is to see if such a change in working methods can contribute to pupils' ability to solve arithmetic problems.

The basic intention underlying the design and content of the lessons in the classroom study was that the pupils should be able to speak their own language, carry out different actions, and vary their perspective on arithmetic problem solving and the problems posed. Sixty-eight lessons were observed and recorded

on audio tape. The empirical material includes all the documents produced by pupils while solving the problems, comprising written stories, drawings and arithmetic calculations. The pupils who participated commented on the lessons in interviews and written reports. The class teachers have also given their comments on the lessons and the pupils' learning.

The results show that the pupils in the participating classes solved the problems in the post-test to a greater degree than did the pupils in the control classes. A covariance analysis shows that the difference between the two groups was statistically significant.

Pupils' understanding of a problem depends on their prior experiences, the problem situation and the specific problem. They have a preconception of a given problem which gives them a diffuse overall understanding of it. Thereafter they differentiate within the problem and while solving it they refer to different parts of the content. When the pupils relate the problem's parts to one another and integrate them to a composite whole the problem takes shape such that they understand and conceptualise the problem. This understanding is seen in their orientation and approach to the problem and in their conception of the problem.

It is found that pupils can have two orientations, in which they have different goals or intentions for solving the problem.

Taken-for-granted orientation: Pupils have a product-intention; they want to give an answer to the problem.

Open orientation: Pupils have a process-intention; they want to seek an answer to the problem.

Pupils approach the problem in four qualitatively different ways.

Operand approach: Pupils focus on the numbers; they estimate a numerical answer to problems and do not carry out calculations.

Procedure approach: Pupils focus on numbers and operations; they perform a calculation to reach an answer.

Hypothesis approach: Pupils focus on all parts of the problem's content, and try to see how they are related. They do not, however, relate the numbers given to the relevant content of the problem.

Gestalt approach: Pupils focus on all the parts of the problem and on the relationships between them.

Pupils with a taken-for-granted orientation have an operand or a procedure approach. They see only the problem's surface and problem solving means that they have to give a numerical answer or perform an arithmetic operation. They apply well-known methods to the solution and solve the problem according to a fixed pattern. Pupils with an open orientation have a hypothesis or a gestalt approach. They go into the problem in depth and problem solving means that they relate the parts of the problem's content to one another. They are captivated by the problem solving process; they pose hypotheses, try different alternative solutions, and can vary their perspective.

A pupil's orientation, approach to and conceptions of the problem determine how the problem solving process takes shape. The process itself is characterised by the three components direction, reference and movement which in a dialectic relationship determine the outcome of the attempt to solve the problem.

Pupils' different ways of experiencing arithmetic problems and problem solving are formed in the meeting between their own ideas, the classroom situation, and the content of the lesson. The problems which pupils meet have a meaning for them and the problems present themselves in the light of the pupils' earlier experience. Teaching of arithmetic problem solving should therefore to a greater extent give the pupils the opportunity to form arithmetic relationships and discover mathematical structures with a basis in their own experiences.

- **Löthman, A.** (1992). *Om matematikundervisning – innehåll, innebörd och tillämpning: en explorativ studie av matematikundervisning inom kommunal vuxenutbildning och på grundskolans högstadium belyst ur elev- och lärarperspektiv. [On mathematical education – content, meaning and application:* an e*xplorative study of mathematical education in municipal adult education and upper compulsory school from the perspectives of students and teachers.]* (Uppsala Studies in Education, 40.) Stockholm: Almqvist & Wiksell International.
  University of Uppsala, Uppsala Studies in Education. *Supervisors*: S. Lindblad, C. Gustafsson (Uppsala).

Abstract

On Mathematical Education – Content, Meaning and Application
This is an explorative study of mathematical education in municipal adult education and in the compulsory school, senior level, elucidated from students' and teachers' perspectives.

Background
As a teacher of mathematics I have several times thought about the students' real understanding of mathematical concepts. What is their own thinking and what is a reproduction of the teacher's thoughts? Students solve the problems correctly but on which level is the comprehension? Have adults, who renew their studies in mathematics, any different strategies in order to solve the problems? In what

way can education contribute to stimulate students to mathematical reflections of their own and how can students themselves contribute in this process? For a long time I have sought for answers on these questions, but the answers were rather unstructured and fragmentary, which was very unsatisfying. In accordance with the Swedish national curriculum, mathematical teaching will proceed from students' experiences and needs, and prepare them for the role as adult citizens (Lgr 80, curriculum document). Mathematical teaching will contribute to develop their personalities and strengthen their roles as citizens (Lvux 82, curriculum document, adult). The main point in mathematics emphasizes that problem solving will be a very important part of mathematical education and will increase students' ability to discuss and judge a mathematical solution. To communicate mathematically is to express oneself using language of mathematics. That is an important part of the education according to the curriculum.

Aim

The object of research in this study is mathematical education and its actors. The aim is to describe students' and teachers' conceptions of mathematical education in connection with a concrete educational course. In these conceptions I try to find patterns and structures and catch their importance for the actors and the education.

The main aim of the empirical study in the concrete situation in the classroom is to investigate the actors' conceptions of the education. In the study I also intend to identify the students' knowledge of mathematical concepts used outside school. The participants in my study consist of students and teachers in two classes, one from secondary school (16 years old, grade 9) and one from municipal adult education (first part, stage 1). Both classes are studying the equivalent mathematical courses on this occasion.

How to catch the phenomenon of mathematical education viewed from the students' and teachers' perspectives? Where and with which instruments would I identify different aspects of the mathematical education? First I chose to study literature about classroom research. There I found research schemes and research models concerning education that could be the basis of my work (Dunkin & Biddle, 1974; Gustafsson, 1984). In classroom research I also included approaches like student and teacher thinking (Wittrock, 1986; Clark & Peterson, 1986) and phenomenography (Marton, 1981), which in some way represent another kind of classroom research. On the individual level I was interested in research into communication, interaction and constructivism in order to understand students' and teachers' strategies and intentions in mathematical teaching and learning.

Research in factors' perspectives

The origin of my approach derived from classroom research, where the use of observations was a frequent method. The researcher sought an answer to the question "What is teaching like when it is effective?" Those who asked that question did not do it to achieve cognitive clarity to understand what was going on in classrooms and why it happened the way it did. Instead they wanted to know how teaching could be improved. Many kinds of different analysis schemes were used, where Flanders' observation scheme (1970) belongs to one of the most well-known. This tradition of research is called the process-product research and was an ordinary approach in the USA from 1920 and onwards. This approach was criticized in the middle of the 1970s because of its studying only observable phenomena, for example students' and teachers' behavior and students' achievements (Peterson, 1988).1t was presumed that there was a direct connection between the students' achievements and the teachers' behavior. Also the content of teaching was generally neglected.

A new perspective

The emerging interpretative research maintained that students' conceptions and interpretations of the education are the basis for learning and success. In the qualitative research, where interpretative research is included, the researcher is interested in the ways theory and hypotheses can best be generated. The purpose is to understand, not to prove! An analysis of even a single, unique case may give knowledge of great value like theoretical concepts to be used in other situations. The qualitative researcher makes use of intensive analysis of the data for the purpose of generating new concepts and theories. He has to study a very limited but carefully selected number of subjects or informants (Ahlström, 1990). The connection with process-product research and interpretative research is illustrated in a figure by Ahlström (1990).

In the middle of the 1970s the interest in classroom research was extended to special studies of the teacher as an extension of the process-product research (Clark & Yinger, 1977). Zahorik (1980) thought that the interest could be seen as a reaction to the earlier research. This research had introduced universal characteristics for an effective education, but now knowledge and comprehension of the education itself were needed (Erickson, 1986). The process-product research presented an image of the teacher as a technician or operator, who executed skilled performances according to prescriptions defined by others. However, with the breaking of cognitive psychology in the 60s with the interest in higher order mental processes, another conception of the teacher appeared. The teacher was considered as a clinician and decision maker (Ahlström, 1990). This is one of the foundations in the approach called teacher thinking. The image of the student in process-product research was one of a passive recipient of instructional treatments (Doyle, 1977). But the representatives in the cognitive psychology were interested in how the students understood the instruction and interpreted their tasks. Now the student is conceptualized as an interpreter, sensemaker and constructor of knowledge (Ahlström, l990). Research in thought processes proceeded from the process as a neutral concept, where actors' understanding and their conceptions of thought process were not observed. In phenomenography, however, researchers try to describe, analyse and understand how people think about specific phenomena. They do not claim to be describing how reality actually is, the first order perspective, but rather how it is perceived by a person, the second order perspective (Marton, 1981). The phenomenography researchers study the actors' understanding and conceptions of a phenomenon. The goal of the analysis is to make descriptions of the variation in the actors' conceptions of the phenomenon.

Communication, interaction and constructivism as parts of teaching and learning

Communication in everyday life situations has been studied within a microsociological and especially an ethnomethodological tradition. These studies aim at penetrating the details of the communicative patterns underlying social interaction (Garfinkel, 1967). A basic assumption is that the social world essentially and continuously is achieved by members who share a common sense knowledge about it. Mehan (1979) has studied the social organization of classroom communication as it occurs in natural classroom events. Through in-depth analyses of video- and audiotapes of student and teacher interaction he sets out to locate the organization of classroom lessons and the interaction. He found for example that the lesson is built up of sequential and hierarchical phases, like opening, conducting and closing the lesson.

Teaching and learning are parts of the same process, according to Pask' s conversation theory. So they ought to be described in the same terms (Pask, 1976). Lindström (1983) discusses concepts like learning strategies and learning styles, where learning strategies are attached to the "operation" in a special subject, for example mathematics. Learning styles are attached to the cognitive aspects like perceptions, expectations, memories and understanding.

In an educational process, which is derived from a constructivistic view, the actor himself constructs and shapes ideas and ways of thinking. The teacher does not think he is bringing out knowledge to the student. Instead he is stimulating active construction and turning into knowledge. Research with a constructivistic attitude is usual in scientific subjects, for example in studies from Lybeck (1981), Halldén (1982), Renström (1988) and Bergqvist (1990).

Research in mathematical education

Romberg & Carpenter (1986) discuss and describe different studies of teaching and learning in mathematics. They establish that in a number of studies in the early 1970s the teacher's actions during the lessons were in focus. The content and the students' actions have been elucidated to some extent. However, since the later part of the 1970s the cognitive science in education and in research has attained new important aspects of teaching and learning in mathematics. Weaver (1987) thinks that there are four areas of mathematical educational research in focus now:

Research in mathematics is much more likely to focus upon individuals than upon groups. That is to say learning is an individual phenomenon.

The researcher looks beyond observable performance at so-called behavioural objectives for the data base. It indicates that internal cognitive processes are acknowledged.

Research in how young children invent or construct their own mathematical knowledge.

Research in metacognitive aspects of the problem solving processes.

The application of cognition, constructivism and metacognition brought another type of mathematical problems. The early drill and "back to basics" movement did not take advantage of students' ability (Schoenfeld, 1987). In 1980 there was a declaration in the USA that problem solving ought to be in focus in mathematical education. The same thing also happened in other countries. In Sweden the curriculum documents, Lgr 80 and Lvux 82, have problem solving as an integrated plan in mathematics. Schoenfeld (1991) calls attention to the fact that this change in mathematics needs more than merely understanding the terminology of the plan, it calls for being able to communicate mathematically, to express oneself using the language of mathematics. Mathematics could be looked upon as a communication subject. In this problem solving movement you can also find contributions from Polya (1945).

Mathematical research in Sweden has developed out of quantitative measurements of students' mathematical knowledge (Svensson, 1969; Husén, 1967a, 1967b; Kristiansson, 1979; Murray & Liljefors, 1988). The investigations were carried out with many students, who made some written tests (Kilborn & Lundgren, 1973). For example the qualitative research in mathematics in the 1980s deals with young students' thinking (Neuman, 1987) and with the school and everyday mathematics (Unenge & Wyndham 1988; Wistedt, 1990). This can be seen as a part of ethnomathematics (D'Ambrosio, 1985), where the researcher is interested in the mathematical understanding on an individual level and in its context.

The pilot study

The purpose of my pilot study in 1988 was to examine practically some methods and techniques. In the teacher thinking approach the use of interviews according to the stimulated recall technique was frequent in empirical studies. Consequently this technique ought to be appropriate to my study. I decided to find it out in different groups in mathematics. The stimulated recall technique was originally used by Bloom (1954) and consists of replaying a videotape or audiotape of teaching episodes to enable the viewer to recollect and report on his or her thoughts and decisions during the teaching episodes. In my study I used the following methods and techniques (Löthman, 1990).

Recording lessons with two video cameras. One camera with wide-angle objective lens in front of the class and one camera at the far end of the classroom.

Questionnaires for students and teachers.

Stimulated recall interviews with students and teachers.

The main study of students' and teachers' perspectives on mathematical education

The main study started in May 1989 and was of explorative character and consisted of two empirical parts, partly one study which dealt with the selection of a mathematical section, partly one study that caught the concrete education and elucidated the situation from the actors' perspectives. To define my selection of mathematical sections and mathematical problems I raised the following criteria.

The content of the mathematical problems should be descended from the students' experiences.

The problems should contain text and have alternative solutions.

The problems should demand more than one calculation in order to get the answer.

The results of the problems should give a possibility of discussion, analysis and valuation.

These criteria have a frequent representation in problem solving in different moments. In my looking for an adequate mathematical section I examined three sources out of my criteria, namely Standardized Tests, 1965–1985 (Henricson, 1987), mathematical problems from the IEA investigation (The International Association for Evaluation of Educational Achievement) (Murray & Liljefors, 1983), and some textbooks in mathematics. I chose these three sources because they could give information on the official goals of the education from an international and national perspective for example. In the working up I found by way of example that problems of geometry had a lower solving frequency than problems of percentage calculation. Were the problems of geometry difficult to deal with in point of language? My analysis showed that the problems of percentage calculation had a simpler language and a more everyday content than the problems of geometry. Unenge (1990) shows in a limited quantitative study of articles in a Swedish national newspaper that concepts like percentage, area and volume are the most frequent mathematical words in the articles (24%). I maintained from the results of my analysis that percentage problems have a more everyday character and are connected with actors' everyday knowledge. So I was going to choose these kinds of problems in my study of education.

The empirical study of education

The study was carried out in a small Swedish town with four compulsory schools, one gymnasium and one section municipal adult education. The participants in my study consist of students and teachers in two classes. One class from grade 9 (16 years old) with 13 students, seven girls and six boys, and one class from municipal adult education first part, stage 1, with 7 students, two women and five men. The adult students had an average age of 33 years and the men had worked as builders, mechanics, transport workers and the women had worked in hospitals and restaurants. As I was interested in how students and teachers conceive the education I chose groups with different backgrounds and ages. Both classes were studying the equivalent theoretical course in percentage calculation for the compulsary school, senior level, and the education had this content:

A repetition of concept percentage from basic.

Calculation with raising and reduction of prices expressed in percentage.

Calculation with problems where the percentage numbers are asked for.

Practical application of percentage calculation like value added tax, interest, discount, rates and taxes.

Procedure

I planned my study on the basis of my experiences from methods and techniques in the pilot study and my received knowledge from research literature. The interviews had appeared suitable according to the stimulated recall technique and I intended to use that together with questionnaires to students and teachers. I also planned to study and analyse the content of the videofilms. Consequently I was going to use partly a perspective from inside, partly a perspective from outside in order to catch the actors' conceptions of the mathematical education.

In grade 9 I videofilmed seven lessons, together 280 minutes, and in the adult group I videofilmed six lessons, together 270 minutes. This camera was turned to the students. Another camera was turned to the teacher and videofilmed during the teacher's work at the blackboard, in grade 9 about 100 minutes and in the adult group about 150 minutes. In connection with the lessons I used the stimulated recall technique and interviewed the teachers and the students. The students were interviewed individually or in groups.

Table 1. Account of students' interviews

	Number of interviews	Number of hours
Grade 9	22	10.5
Adult group	29	14.0
Total	**51**	**24.5**

The interviews emanated from the actors' descriptions of what they had seen on the videofilms. Then I gave complementary questions from their descriptions about the mathematical content and the lesson. I also used the students' textbooks in mathematics. The teachers were interviewed in the same way 2.5 hours each, total 5 hours. All interviews were transcribed verbatim. The students had to answer some questions about the education at the end of every lesson. Before the lesson the teachers had to account for their planning and afterwards they had to write down the results from their perspectives.

Findings

After successive workings up and analyses of the interviews I got a number of descriptions of the conceptions of the education in mathematics. The adults' discussions of mathematical knowledge, particularly percentage calculation, touched upon their experiences from private lives and work. It dealt with money like bank loans, wages, prices of dwellings and living costs. The boys in grade 9 on the other hand had quite different conceptions of percentage calculation and where you can find it in everyday life. They were talking about finances like shares and share prices and about saving in banks. The girls in grade 9 moved within a limited mathematical sphere with more immediate commonplaces. Their conceptions of percentage calculation in my study for example circulated around the purchase of clothes and sales. Perhaps the social background of the students in the current study had a special role here. The boys came from homes with traditions of education contrary to the adults and the girls, who in common were without such traditions. I maintain that the boys searched for the mathematical problems. The girls in return caught the mathematical problems that appeared to them.

What use is the knowledge of mathematics to the students? In accordance with Mellin-Olsen (1987) I identified two major rationales driving for school learning. The adult students in my study thought that their knowledge of mathematics could be used in coming studies. They were orientated towards an instrumental rationale. Young students in grade 9 answered that their knowledge could be used in the future in their work and hobbies. They had a social rationale. It is a rationale which says that knowledge has an importance beyond its status as school knowledge (Mellin-Olsen, 1987). Both the rationales were of course sometimes integrated.

Teachers and students were aiming at teaching and learning mathematics, but the meanings of these activities were different for the actors. It appears to me that this was one of the most representative and essential aspects of my empirical study. The meanings of the teaching and learning activities were pointing in two directions. One was aiming at comprehension and another was aiming at procedure. Naturally these two directions were not quite separate. They occurred side by side. But the actors showed some distinctive traits.

Table 2. Comprehension and Procedure orientation

	Grade 9	**Stage 1 Adult**
Student perspective	Procedure	Comprehension
Teacher perspective	Comprehension	Procedure
Lgr 80, Lvux 82	Comprehension	Comprehension

The descriptions elucidated some differences in the meanings and the importance of education. It appears to me that the differences are due to the backgrounds and the experiences of the actors. Young students desired to solve many problems during the lessons. It was important to them. The teacher knew the content and the objective of the curriculum and tried to discuss the mathematical problems with the students. The adult students had partly their previous less successful studies, partly their experiences from different occupations. I suppose that these experiences convinced them of the necessity of understanding and the number of counted problems was not so important to them.

Result pictures

In the phenomenographic approach the descriptions of the phenomenon from the second order perspective are the result. In my case the actors gave expression to different conceptions, which I then described in the form of patterns and structures. These descriptions had been put into a number of result pictures, showing the teaching and learning as an entirety.

The four pictures are:

Mathematical traditions, consisting of the conventional handling of mathematical problems in relation to the students' experiences.

Mathematical strategies, consisting of the students' way of understanding, reflecting and solving mathematical problems.

Mathematical reasonings, consisting of the students' way of discussing, analysing and judging mathematical information.

Mathematical applications, consisting of the students' way of understanding and practising mathematical concepts in situations outside school.

These areas are not isolated, instead they complete and support each other and are important components in the education. They catch some pictures of the complicated situation in teaching and learning. Mathematical traditions are an integral part in all the pictures. In my two groups I found different kinds of traditions. In the adult group the students and the teacher both agreed with strong rules and formal dipositions of problems. Höghielm (1985) confirmed the teacher-ruled education in adult groups. In grade 9 the students also wished the same thing about rules and dispositions. The teacher in return thought that an informal and creative calculation was sometimes preferable, but the students did not accept that. Mathematical strategies showed a lot of procedures. Some of them were due to the adult students' practical experiences of calculations and their earlier education. The strategies also revealed lack of mathematical knowledge in young students in grade 9. For example in a mental calculation problem the students had to count 30% of 40 kr. The following discussion was carried out in the interview.

Karin: Yes, I take 3 multiplied by 4. It is 12.
Int: But the naughts, then'?
Karin: They cancel out!
Int: What do you mean by that?
Karin: Yes, one naught cancels out the other!

There were similar problems with the strategies in the adult students. They have been taught percentage calculation earlier through dividing by 100 and now they had to use decimal fractions in the calculation like 5% = 0.05. In the beginning they were rather scattered in relation to new solving strategies. It appeared as if they used the old strategies as a mental calculation and the new strategies in the book at the same time. The teacher tried to convince them that the decimal fraction method was very important for mathematical studies in the future. The following problem was given as a mental calculation during a lesson.

How much is 25% of 300 kr?

At the interviews the students had to explain their solving strategies. One of the adults argued in this way.

"Oh, it is 25 kr on every one-hundred note and there were 3. It will be 75 kr."

The teacher in the adult class asserted that the students would probably have to solve this problem according to the decimal fraction method. He did not mention anything about the "practical" method. With the aid of the interviews and my studies of film sequences from lessons I could make a number of strategies visible. Some of them exposed lack of mathematical understanding of percentage concepts and the using of decimal fractions. Other strategies proved to be practical, but not so useful and desirable in the perspective of mathematical development. These differences and the lack of mathematical understanding were not discovered during the lessons as the students should often only give the right answer. Lybeck (1986) declared that there are two kinds of thought strategies, one called the A-strategy, which is general, and one called the B-strategy, which is specific. In my case the A-strategy is represented by the decimal fraction methods, while the B-strategy is represented by the method with 25 kr on every one-hundred note. Both methods were useful here but the A-strategy led to a development for coming studies, according to the teacher. The B-strategy had a real connection for the adult students. Mathematical reasonings contained important information about the actors' way of reasoning more or less enthusiastically about a problem and its solution. The reasoning exposed a process which had different meanings for the participants. The adult wished the whole course to be clear, while the young students inclined towards a rapid and routine model to solve problems. These quotations illustrate the differences.

Student in grade 9.
First my teacher will instruct me, then I will have to solve the problem, then he will ask why. Instead of telling me directly how to do it!

Adult student.
I must know and understand my actions in solving problems. Previously in secondary school I did not understand why I was solving the problem in that way. But now things are getting clear!

Mathematical applications in a concrete situation elucidated the importance of experiences. The adult had a knowledge of the percentage concept outside school, for example interest, but they were uncertain about calculations in practice sometimes. The young students, particularly the girls, had a vague knowledge of interest for example. They did not know how the bank could pay interest to them every year. Where did the money come from? The following question was given to the students in grade 9.

What does the bank do with the interest?

Jessica: I don't know! They pay it to us?
Int: But where does the bank get the money?
Jessica: From the state...? I have never thought about it.
Int: What do you think?
Jessica: The value of money rises, perhaps!
Int: Sorry! I think it is quite the contrary!
Jessica: I have no idea!
Int: But if you must borrow money?
Jessica: Yes, I will have to pay high interest!
Int: Where does the bank get the money?
Jessica: Oh. I see. From the person who pays high interest. I get money from others!

The content in this interview showed the young students' difficulties of connecting mathematical problems with concrete situations outside school. It appeared to me that the rule-based mathematical education in school in my study had lost contact with the world outside school. The practical function of the knowledge had not been followed up in the education. The teacher admitted that this was a shortcoming and also the textbooks did not talk about this very much. But the teacher hoped that the student would learn this as adults. The adult students had a knowledge of these concepts through their experiences and disposed and used strategies to solve the mathematical problems in practice.

Discussion

The phenomenographic approach gave the opportunity to study the mathematical education of students' and teachers' perspectives. The stimulated recall technique was an adequate instrument to catch the perspectives and the study of three mathematical sources introduced aspects of the mathematical problems and mathematical education during approximately 25 years. I also tried to penetrate the phenomenon of teaching and learning in mathematics by using different constellations of groups. I maintain I got a covering picture of the education from the actors' perspectives by comparing students in grade 9 to adult students in stage I and the teachers in the two groups. I also got information about the education by comparing boys and girls in grade 9, older and younger adult students and the students' social backgrounds to their attitudes and conceptions of mathematical knowledge. I have classified the results of the described conceptions in four pictures, which elucidate some important patterns and structures in the mathematical education. The four pictures are the basis of some other findings from the actors' perspectives made in an ethnographic approach. The findings focus on two directions in the actors' conceptions of teaching and learning. The study introduces two directions, namely *comprehension* direction and *procedure* direction. The analysis also gives an exposition of two different ways to learn mathematics relating to gender. The boys *searched* for the mathematical problems and the girls *caught* the mathematical problems that appeared to them. This study has focused on some different pictures of learning and teaching in mathematics from the actors' perspectives. I hope that my results in some way or other may contribute to a more "dynamic" education in mathematics in Swedish schools.

- **Wyndhamn, J.** (1993). *Problem-solving revisited: on school mathematics as a situated practice.* (Linköping Studies in Arts and Science, 98.) Linköping: Linköping University.
  Linköping University, Institute of Tema Research.

Abstract
The general interest behind the present work is to contribute to an understanding of human cognition in context. More specifically, the empirical research reported focuses on how pupils define and deal with problem-solving in what for them is a regular school situation. The relationships between problem-solving and contexts – concrete settings as well as abstract framings – are explored. Thus, cognition is studied within a sociocultural perspective and as embodied in communicative practices within institutional settings.

The empirical material reported has been collected in a series of naturalistic experimental and quasiexperimental investigations in which pupils in different grades of the comprehensive school have solved mathematics problems under varying conditions. The variations introduced have concerned such dimensions as (i) manipulations of the contextual embeddedness of problems in terms of headings provided, formulations of word problems, composition of tasks on work sheets, and the lessons during which identical tasks have been attended to, (ii) the tools and resources (computer, calendar) that have been available to pupils when solving problems, and, finally, (iii) whether students have been working alone or in groups.

The results demonstrate the fundamental, yet subtle, ways in which different contextualizations of problems result in variations in interpretation and the relative difficulties pupils have in dealing with them successfully. Problem-solving

in the mathematics classroom is best understood as a simultaneous coordination of several levels of activity. There are intricate rules for how word problems are to be interpreted and what sorts of semiotic principles are relevant and can be relied on when coordinating what is said with an external reality and with mathematical notations and operations. Similarly, there are many levels of context in the institutionalized setting that offer structuring resources that provide suggestions for how to interpret tasks. It is also argued that mathematics teaching often results in a dilemma which is paradoxical in nature; in the attempts to utilize problem-solving as a vehicle for promoting competence that is decontextualized and abstract, mathematical reasoning itself becomes the dominant context for making sense of the tasks to be solved.

- **Chen, X.** (1996). *Quality schooling with limited resources: an international comparison of mathematics and science education in China, Korea and Hungary.*
  (Studies in Comparative and International Education, 37.) Stockholm: Institute of International Education.
  Stockholm University, Department of International Education.

Abstract
There have been growing concerns on achieving quality schooling with limited resources in many developing countries. This study compares China (People's Republic of China) with Korea (Republic of Korea) and Hungary in Mathematics and Science Education. The general aim of the study is to examine the patterns of relationships between student achievement and background factors between China and the two other high-achieving countries that have limited resources.

The target population was 13-year-olds (born in the calendar year 1977) in the three countries. In total, 10,071 students of 373 schools were involved in this study. Half of them were assessed in mathematics, while the rest were in science. A student questionnaire, a school questionnaire, and a national questionnaire were administered. Data was collected in 1991 in the framework of the second International Assessment of Educational Progress Project (IAEP). Multivariate analysis was conducted through LISREL modeling.

It was found that Chinese students achieved higher in mathematics and lower in science than their counterparts in Korea and Hungary. The weakest curriculum topic in China was data analysis, Statistics and Probability in mathematics and Biology in science. Students in the three countries were all positive towards mathematics and science. However, the Korean students were more critical towards mathematics and science learning.

Findings from this study show that parents in the three countries were very supportive toward their children's school learning. The number of books at home, and the number of siblings which a student has, had significant relations with student achievement. The supportive attitudes of parents (such as encouragement and interest in school learning) were more influential on achievement than family assistance with student homework. Furthermore, student leisure reading exerted positive effect on educational outcomes across the countries. However, TV viewing had a negative relation with student achievement in mathematics and science.

At school level, it was found that availability of a school library and laboratories was more influential on achievement than possession of computers. Large schools were usually better equipped than smaller schools. If school equipment was the same, smaller schools had better results. Students in smaller classes achieved better than in larger classes, if equipment was constant. Concerning teaching practice, it was observed that mathematics and science were mainly taught through teachers' presentations. The status of educational provisions differed markedly among the countries under study. The schools in Korea had more physical equipment than their counterparts in China and Hungary, whereas more literary resources and better trained teachers were available in Hungarian schools. However in China, educational resources were concentrated in only a small proportion of schools that were both well-equipped and well-staffed. As a result, the between-schools difference in achievement was larger in China than in Korea and Hungary.

This study has implications for educational policy and practice in China, as well as further studies in this field.

- **Dunkels, A.** (1996). *Contributions to mathematical knowledge and its acquisition.* (Doctoral thesis, Tekniska högskolan i Luleå, 202D.) Luleå: Luleå University, Department of Mathematics.
  Luleå University, Department of Mathematics. Supervisor: L-E. Eriksson
  Opponent: M. Niss (Roskilde).

Abstract
This is a thesis in mathematics with didactical emphasis. The interest of didactics of mathematics is in mathematics in relation to how it is learned.

The thesis comprises three parts, A, B, and C. Part A reports in three papers from research in mathematics within potential theory. Problems with electrical conductors form the historical background, where Gauss found mathematics interesting in its own right. He generalized the concepts and laid the ground of

a rich mathematical area of research with many applications within other parts of mathematics. In Part A further generalizations are made.

In seven papers in Part B reports are found from action research, or developmental research, in the classroom reality from primary to tertiary level. Fieldnotes are the source of data. In Part B one finds the basis of the ideas for Part C.

Part C describes a project in a class of first year engineering students with the mathematics teaching completely devoid of exposition by the teacher. In the classroom the students worked in cooperative groups of four to the group with the teacher as adviser and discussion partner. One of the main questions was how this form of learning would affect the students' results from a cognitive as well as an affective perspective. The different methods of research used all point to the fact that the influences of the project have been positive. During the project a new theory was developed for how the "fumbling" and not quite straightforward acquisition of mathematical knowledge may be described. The theory is useful for teachers as well as students and is in accordance with the way researchers in mathematics work. Thereby, in a natural way, this theory connects the three parts of the thesis.

- **Ekeblad, E.** (1996). *Children – learning – numbers: a phenomenographic excursion into first-grade children's arithmetic.* (Göteborg Studies in Educational Sciences, 105.) Göteborg: Acta Universitatis Gothoburgensis. University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract
Arithmetic is one of the major areas of learning that children encounter in first grade. It is evident that learning to add and subtract does not come easy to all of them, and it has been observed that basic arithmetic may have a strong influence on children's self-definitions as learners, and their understanding of what learning is. It has also been suggested that when mathematics difficulties arise they stem from the child's adoption of an approach to addition and subtraction where problems are solved by unit-by-unit counting. When children do this there is a risk that they never come to experience numbers as whole-to-parts relations, which is the fruitful way of approaching basic arithmetic.

This dissertation examines on the one hand the scope of variation in first-grade children's ways of experiencing *learning* in the domain of arithmetic and on the other hand the scope of variation in their ways of experiencing *numbers* in the context of first-grade arithmetic. The material for this phenomenographic analysis was derived from a one-year study of three first-grade classes using some computer software developed for the purpose of enhancing the whole-to-parts aspects of numbers.

The main distinction in terms of the children's ways of experiencing number *learning* was between number learning as something that comes easy, as a gift (or not at all), and number learning as something that comes through work. In either case there was also a variation in whether, and to what extent, the learning self was seen to have an active role in number learning.

Three dimensions, each constituted by two contrasting poles, were discerned with reference to the children's ways of experiencing 'numbers' in the context of learning. Numbers as experienced by these children varied in terms of their organisational integration of the poles of 'structure' and 'sequence', they varied in terms of their presentational 'nebulousness' or 'definiteness', and they varied in terms of whether they were oriented to as phenomena of 'authority' or as 'autonomous' phenomena. These three dimensions could also be viewed as an ontological, an epistemological and an ethical aspect of numbers, and part of the difficulty with number learning may stem from internal discrepancies in the available ways of experiencing numbers and number learning.

- **Engström, A.** (1997). *Reflektivt tänkande i matematik: om elevers konstruktioner av bråk. [Reflective thinking in mathematics: on students' constructions of fractions.]* Lunds Universitet. (Studia psychologica et paedagogica, 128.) Stockholm: Almqvist & Wiksell International.
  Opponent: O. Björkqvist, Åbo Akademi, Finland.

Abstract
This thesis deals with students' constructions of mathematical knowledge in the case of fractions from a radical constructivist perspective. Fractions are the first more abstracted mathematics met by students. Even if they have a rich experience of sharing and partitioning things in their everyday life, dealing with fractions often gives rise to difficulties in school.

The thesis consists of two parts. The first part constitutes a discourse on radical constructivism in the research field of mathematics education. The second part is an empirical study wherein we discuss earlier empirical investigations of students' achievements on fractions and our own empirical inquiry.

Constructivism has become one of the major influences on contemporary research in mathematics education. In our discourse we give an account of its development in the light of three main changes in the philosophy of science and mathematics during the 20th century.

Radical constructivism is a theory of active knowing. Knowledge does not depict or represent an experiencer-independent reality. It serves to organise the subject's experiential world. Our world is not an unchanging independent struc-

ture, but the result of distinctions that generate a physical and a social environment to which we adapt as best we can. Reality is a reality created by the subject.

Jean Piaget, in his research programme of genetic epistemology, offers two accounts for the development of human understanding. His structuralist account is concerned with the organisation of human knowledge in a system of an overarching structure. His constructivist account focus on how these structures develop through a process of equilibration. Equilibration is a process of self-regulation leading to better structures from existing ones.

Learning rational numbers remains a serious obstacle in students' mathematical development. When looking at the application of rational numbers to real-world problems from an educational point of view rational numbers appear in numerous forms. A full understanding of fractions seems to require exposure to numerous rational notions. Some Swedish and foreign empirical studies are reviewed. Former attempts to elaborate hierarchical models and learning levels seem to have failed.

The aim of the empirical study was to investigate the schemes that the students develop when working with fractions. We strive to build models of students' mathematical knowledge in terms of coordinated schemes of actions and operations in a Piagetian sense. The method used is the constructivist teaching experiment. It has been a long term interaction between the researcher and two groups of students, A (n=24) and B (n=19), in the intermediate grades during four years. We have made classroom observations, written tests, teaching episodes and clinical interviews with six students from each group. The schemes have been analysed and interpreted in terms of part-whole and part-part relations, N-distraction and differentiation between spatial and logico-arithmetical structures. Students' development of mathematical knowledge is achieved in an interplay between mathematical and contextual senses of a problem. The task of the teacher is to organise activities in the classroom as part of a conscious mathematisation of students' experiential reality.

- **Sandahl, A.** (1997). *Skolmatematiken – kultur eller myt?: mot en bestämning av matematikens didaktiska identitet. [School mathematics – culture or myth? Towards a determination of the didactic identity of mathematics.] (Linköping Studies in Education and Psychology, 51.)* Linköping: Linköping University. Linköping University, Department of Education and Psychology.

Abstract

Mathematics is a sociocultural phenomenon. In educational contexts, no account has been taken of this; instead, learning has focused on the formal language.

In this study, pupils' and students' conceptions of the school subject Mathematics are analysed. The ability to describe what takes place in teaching contexts is of importance in didactic research. This regards especially people's conceptions of how they learn and what the content is. In this thesis, a description is given of people's conceptions of a specific content and how a new aid, the pocket calculator, affects the teaching of mathematics.

Qualitative research makes it possible to describe, interpret and understand different phenomena. With the methods used in qualitative research, reflection in the analysis must take place from different perspectives in a lengthy process. An increasingly accepted conception is that the triangulation of methods reinforces validity. Validity is also reinforced in the thesis by virtue of the fact that students and teachers were able to participate in the process over several years.

The first two empirical studies, carried out over a period of seven years and in which 7,000 pupils and 900 trainee teachers were involved, deal with conceptions of mathematics in school. Although the descriptions originate from different points in time, they describe more or less the same mathematics teaching.

In the third and fourth studies, pupils' conceptions of the pocket calculator are studied. The difference between these two studies is that in the third study, the pupils had access to pocket calculators in school after several years of studying mathematics. In the last study, the pupils used pocket calculators from the beginning. This affects the role of the teacher and the methods teachers use in teaching contexts.

I felt that it was important to uncover the pupils' conceptions of mathematics since these conceptions are a significant factor in their learning. Also significant are students' conceptions prior to their option of mathematics or language. The results show that the teaching of mathematics in school has not succeeded in demonstrating the usefulness of the subject or showing connections between different mathematical skills. The technical skills were not linked to each other in any context. Since pupils and students constantly returned to the view that mathematics was about numbers and doing arithmetic, it would be interesting to see whether doing arithmetic could be made easier by using a technical aid, the pocket calculator, and whether it could focus mathematics on something else. I was also interested in teachers' conceptions of the pocket calculator as an aid.

The results showed that the training of skills ought to be given a new content. Skills in interpreting, handling a situation and being able to handle numbers are part of this change.

- **Wikström, H.** (1997). *Att förstå förändring: modellbyggande, simulering och gymnasieelevers lärande. [Understanding change: model-building, simulation,*

and upper secondary school students' learning.] (Göteborg Studies in Educational Sciences, 114). Göteborg: Acta Universitatis Gothoburgensis.
University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract *(translated from Swedish by Swedish Research Council)*
The purpose of this study is to investigate whether students in upper secondary school, by working with models of dynamic systems in a computer environment, can gain understanding and experience of dynamic systems' structure and behaviour. The investigation is also made in order to see whether system dynamics is a tool that can help students with concept formation so as to reach goals that are specified by the upper secondary school's guiding documents, for example "to master concepts needed for understanding ecosystems' structure and dynamics as well as grasping the consequences of the disturbances that can affect ecosystems". The study's secondary aim is to examine whether the students test their own understandings and theories with the help of the models. Non-linearity and feedback are central concepts. Further, the study examines whether the work can build a bridge between the real world and formal mathematics.

An experiment is conducted in the form of a course where upper secondary school students build models of their own in a computer environment and investigate the models' behaviour by changing the initial values, variables, parameters and simulation time.

Data from the experiment are obtained from the students' reports, diary notes, video and sound recordings, interviews, and a follow-up comparative investigation one year after the work in the classroom.

The study shows that the students have acquired verbal and mental tools which can be used for describing and discussing time-dependent systems' behaviour and structure. The students discover that systems may have stable and unstable areas. The interviews demonstrate that the students' view of the concept of a derivative function, and their understanding of the concept of change, have been improved. The comparison group often simplifies non-linear relationships to linear ones, and lacks the concept of feedback.

- **Dahland, G.** (1998). *Matematikundervisning i 1990-talets gymnasieskola. Ett studium av hur en didaktisk tradition har påverkats av informationsteknologins verktyg. [Mathematics teaching in upper secondary school during the 1990s. A study of how a didactic tradition has been influenced by the tools of information technology.]* (Report 1998:05.) Göteborg: Göteborgs Universitet, Institutionen för pedagogik.
  University of Gothenburg, Department of Education.

Abstract *(translated from Swedish by Swedish Research Council)*
The report's first part begins with a summary of our own research work conducted during the 1990s. The background is the introduction of electronic tools in the schools' mathematics teaching, accompanied by changes in method. Access to pocket calculators with high performance has accelerated the development.

For some decades, research in didactics of mathematics has been international in scope, and during recent years it has been oriented toward evaluation of knowledge. Didactics of mathematics in Sweden is described against the international background, also with occasional historical references. A previously presented didactic model for computer-supported teaching of mathematics is adapted to modern IT tools.

At the same time as IT has become increasingly useful, the school subject of mathematics has undergone great changes during the 1990s due to the reorganisation of upper secondary schools. The view of, and evaluation of, knowledge are discussed in a temporal perspective that comprises the period from the line-divided school with examinations until the programme school without examinations. Some theories of learning with importance for mathematics are treated.

Teachers, as both new and experienced practitioners of the teaching profession, have a need to connect theories of learning with theories of teaching. This is based on the fact that theories must be applied in the practices of teaching so as to acquire legitimacy as instruments for development. A selection of theories with importance for teaching mathematics are discussed. The introduction of information technology constitutes a problem of integration, since it affects the work of teachers in several areas. A new orientation of methodology and evaluation is clearest. The report considers new possibilities, giving examples of methods. Special problems of language and interpretation that result from teachers' and students' work with IT are presented with examples.

IT is an infrastructure of increasing significance for society. The schools' task of meeting modern needs for society and the individual is discussed. Mathematics teaching makes use of computer support, but primarily pocket calculators, to a growing extent. An account of benefits and risks for the subject's presentation and the individual's development of concepts and skills in the subject concludes the first part.

The report's second part describes a study of how mathematics teachers' methodology and instruction according to their own understanding have been influenced by the existence of electronic tools. The aim is to investigate *how upper secondary school teachers of mathematics, in their instruction, have been influenced by the access to modern electronic aids.*

Primarily studied are changes in teachers' methodology, instruction materials, and methods for evaluation, as well as how the students' use of the modern tools has affected language, solution methods, and communication ability in the subject.

Starting from a theoretical model of education, three instruments for qualitative method have been employed. In a questionnaire designed with free answers, 197 teachers have commented on IT and mathematics. Some 60 tests in mathematics have been analysed regarding formal properties. A group of 13 teachers, in conversations with argumentation, have discussed four areas that are influenced by IT support: students' description of problem-solving, the teacher's methodology, the mathematics subject's content, and mathematical language.

Teachers give instruction about the use of pocket calculators and computers because this supports students' work in mathematics. Motives for not giving such instruction are that the students already know, that there is no need, or that it takes time from essential parts of the teaching. Computer support is refused for organisational reasons. Methodical use of pocket calculators is avoided when the model distribution is large in the student group. The teachers almost always adapt tests to the fact that pocket calculators are permitted aids. In general, great respect is shown for traditional methods, although new methods may be rather hesitantly considered.

In tests, two types of problems are used, which the report calls imperative tasks and test tasks. Tests display large formal differences between schools and programmes. No common standard exists. The material reveals no formal similarity with national tests. The testing times are invariably short.

Students' solutions with pocket calculators can be hard for the teacher to understand, resulting in deducted points. A re-evaluation of what constitutes an approvable solution may be needed. Outstanding students undergo a conceptual development which is strongly influenced by IT tools and differs from that of earlier generations.

In terms of method, graphical pocket calculators have been very important for the developments during recent years. They allow and support alternative ways of working, but the investigation has found that especially the clever students prefer regular, traditional lessons with the teacher as a conveyor of knowledge. Although the mathematics subject's content has not changed, new methodological approaches are described. The content intended for certain programmes is questioned. Many students' difficulties with mathematics originate in poor handling of language, as regards both Swedish and the language of mathematics. Even clever students need to improve their abilities of communication.

Viewpoints from experienced teachers at upper secondary schools have been collected in the study. Against this background, a change in the schools' mathematics instruction is outlined which, according to the author, would facilitate the activity and solve several of the problems that teachers have described orally and in writing.

- **Åberg-Bengtsson, L.** (1998). *Entering a graphicate society: young children learning graphs and charts.* (Göteborg Studies in Educational Sciences, 127.) Göteborg: Acta Universitatis Gothoburgensis.
  University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract
The aim of this study was to uncover fundamental features involved in the understanding of some commonly used graphs and charts. Aspects related to graphics as knowledge objects were investigated in addition to contextual aspects in a broader sense. A point of departure was taken in the necessity for members of our modern society to be "graphicate". Some characteristics of graphic representations are presented and advantages of their use are indicated. A small number of previous studies on young pupils[1] comprehension of graphs and charts (including investigations of coordinates) are recounted in some detail.

Although a mainly socio-cultural perspective was taken, a "phenomenographic" approach was used with the particular aim of unraveling aspects of the graphs and charts crucial for a well developed understanding. During a series of working sessions in school settings, twelve pupils, 7–10 years old, constructed and interpreted four types of graphical designs, namely bar charts, pie charts, pictorial charts, and line graphs, under the guidance and supervision of the researcher. Video-recordings from these sessions along with the children's drawings constituted the major data.

In the first part of the analysis, principles from the phenomenographic research tradition were followed and an outcome space of different ways of construing each of the four types of graphics was identified. For the first three types of graphics, a main distinction was found between construing the depicted quantities as measurable wholes and as distinguishable, countable entities. A number of other aspects were identified which were also common for these formats. The outcome space for line graphs is particularly interesting, as it (a) differs from the other three and (b) to a great extent turns out to involve the comprehension of coordinates. In the second part of the analysis the identified ways of construing the graphics were looked upon, using the activity system creating them as a point of departure.

Thereby, aspects of the context important for the children's construals are brought to the fore. It is shown that the tools which were provided and used played an essential role as to how the different graphs and charts were drawn and interpreted.

- **Hedenborg, M-L.** (1999). *Cognitive strategies in simple addition and subtraction: process models based on analyses of response latencies and retrospective verbal reports.*
  Stockholm: Stockholms Universitet.
  Stockholm University.

Abstract
The purpose of this thesis is to explore cognitive processes used by children and adults when solving simple arithmetic problems, and to develop process models for describing these processes. The studies in the thesis were based mainly on analyses of response latencies and retrospective verbal reports, separately and in combination. Process models are presented, describing problem solving processes in simple addition and subtraction in children and adults. For children, reconstructive solutions, i.e., solutions where a conscious cognitive procedure is used in the process, dominated. For adults, reproductive solutions, i.e., processes in which the answer to a problem is retrieved directly from long-term memory storage, were more frequent.

The process models presented in the thesis are based on a simple counter model, which is gradually revised and elaborated throughout the thesis. In the original simple counter model for addition, a counter is initially set to a starting point, normally the larger of the addends, and then incremented in steps of one, until the answer is reached. For subtraction, the counter starts on the larger number in the problem and counts down, or on the smaller number and counts up, according to a certain rule. The starting point and the number of steps to count, and steps counted, are assumed to be stored in working memory.

The reconstructive strategies for addition described by children in this thesis included counting up with units of one, counting up with steps greater than one, use of "ties" (problems where the addends are equal) as reference, and idiosyncratic strategies. Consistency in strategy choice resulted in good performances. Reconstructive strategies for children's subtraction included counting up from the smaller number or down from the larger number, in steps of one or in greater steps, use of "ties" as reference, and idiosyncratic strategies. As would be expected, the number 10 was an important point of reference, especially in counting down strategies. The process model for retrieved solutions in adults' subtraction suggested that traces of originally learnt strategies were present in

retrieval. Finally, judged and observed error frequencies in adults' simple subtraction problem solving were compared. Error predictions were more accurate for errors in reconstructive solutions than for errors in retrieved solutions, i.e., in more complex rather than in simpler and more automated tasks.

- **Runesson, U.** (1999). *Variationens pedagogik: skilda sätt att behandla ett matematiskt innehåll. [The pedagogy of variation: different ways of treating mathematical content.]* (Göteborg Studies in Educational Sciences, 129.) Göteborg: Acta Universitatis Gothoburgensis.
  University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract
The present investigation describes the different ways teachers handle the content when they teach fractions and percentages. The aim of the study was to study teaching from a learning perspective.

The study includes five teachers and their pupils in four different schools. Four teachers teach in grade seven, the fifth in grade six. The data consists of audio-recordings of lessons and two interviews with each of the teachers. The teaching has been followed during six consecutive lessons in the respective classes and extends over the first week lessons of the teaching section of fractions and percentages.

When analysing the data, concepts relating to phenomenographic research into learning have been used as analytical tools. Data have been analysed in respect to what aspects of the content the teacher focuses upon, which aspects are left unfocused and which dimensions of variations are introduced when the content is communicated to the pupils.

It can be shown that teachers, when they communicate the content to the pupils, focus or thematize certain aspects of the content and leave others unfocused or unthematized. It can also be shown that variation plays an important role in the teaching process. All five teachers demonstrate an orientation to the content as well as an ability to use variation – although in different ways – when they focus critical aspects of the content taught. By keeping some aspects of the content invariant and opening up for variation of others, a space of variation is constituted. This space of focused aspects and dimensions of variation makes up a potentially experienced object of study. Three different kinds of such objects of study have been identified. When different kinds of objects of study are constituted the pupils have the potential to experience, and thereby to learn, different meanings of fractions and percentages.

- **Häggblom, L.** (2000). *Räknespår: barns matematiska utveckling från 6 till 15 års ålder. [The mathematical development of children aged between 6 and 15.]* Vasa, Finland, Åbo: Åbo akademis förlag.

Abstract

The overall aim of this longitudinal investigation is to chart and analyse how children solve mathematical problems at different ages and to describe how knowledge and skills change during the period when they attend comprehensive school. The target group consists of Finland-Swedish children aged between 6 and 15. When the investigation started, the group comprised 139 children, and in the final assessment 113 pupils participated. The data collection was carried out between 1988 and 1998.

The children's knowledge is assessed in three fields: number sense, arithmetical operations and text-based problems. The assessment is carried out from both a competence- and a communication-oriented perspective. In the competence-oriented assessment, means are calculated when analysing the degree of difficulty of the problems, analysis of variance to describe differences between groups of pupils and regression analysis to illustrate the development of knowledge. In the communication-oriented assessment the children's written accounts are used as the basis for describing variations in the children's arithmetical calculations.

Children's mathematical proficiency varies considerably and stretches over a wide range when they begin school. Most children's knowledge develops during the first school years in accordance with the demands of the curriculum. When the mathematical content becomes more complex, a kind of tie-breaking mechanism comes into effect, for instance due to an increased demand for complex components of thought. More and more pupils run into difficulties.

The pupils' confrontation with the mathematical content reveals differences in the choice of arithmetical method and in the use of strategies. Even at the age of 9, many children are able to give a written explanation of how they count. Children who give wrong answers often use similar counting strategies as children who count right, but various types of arithmetical errors lead to wrong answers. The results show on the one hand qualitatively improving counting strategies with increasing age, on the other hand certain types of error appear during the whole comprehensive school period. The categories of error comprise for instance difficulties in the handling of symbols, the choice of wrong arithmetical operation or the use of a wrong principle of calculation. Even copying errors or omission of figures, wrong placing of the decimal point and linguistic confusion occur.

The investigation shows that the girls are more likely to change than the boys. As regards the number sense variable, there are among the six-year-olds statistically significant differences between the genders in favour of the boys. At the primary school stage the differences are leveled out only to increase once more at the lower secondary school stage. The same tendency is true of the variable text-based problems. With regard to the variable arithmetical operations, the differences between the genders are leveled out more and more during the period of attending school.

The pupils' results were also analysed with reference to their language affiliation, that is, the differences between Swedish-speaking and bilingual pupils. The result with regard to the number sense variable fluctuates between the language groups. At the age of 6, the differences in means are statistically significant in favour of the Swedish-speaking children. The differences are leveled out during the primary school stages only to increase once more in the lower secondary school period. With regard to arithmetical operations and text-based problems, the Swedish-speaking pupils show somewhat higher means than the bilingual children at the lower stages of school attendance, but the differences are later leveled out. The generally predominating Finland-Swedish view that bilingual pupils have difficulties with text-based problems should be modified in the light of the results arrived at. The statistically significant differences that are found between the language groups in favour of the Swedish-speaking children at the age of 6 have already been leveled out at the end of the primary school period. In the higher forms of the lower secondary school, the bilingual pupils even achieve a slightly higher mean in the solution of text-based problems than the Swedish-speaking pupils. The importance of language for the development of mathematical proficiency has many dimensions and leaves room for further studies and continued analyses.

The investigation shows that the children's knowledge develops in different ways in different mathematical sub-fields and at different age levels. The result pertaining to six-year-olds has very little value as a predictor of subsequent development. The division of the pupils into achievement categories, into underachievers, medial achievers and overachievers, indicates that the patterns of change vary from one age level to another. The fact that fewer than 20% of the pupils belong to the same achievement category all through their school attendance period emphasises the importance of teaching for the children's development.

Keywords: mathematics, learning, number sense, arithmetical operations, text-based problems, longitudinal.

- **Lingefjärd, Thomas** (2000). *Mathematical modelling by prospective teachers using technology.* Athens: University of Georgia.
  University of Georgia and University of Gothenburg, Department of Education. Supervisor: J. Kilpatrick (USA).

Abstract
Three studies were conducted to investigate prospective mathematics teachers' understanding of mathematical modelling when using technology to solve a variety of problems. The purpose was neither to verify an existing theory nor to test a priori hypotheses. Rather, the intent was to develop a framework for exploring the students' difficulties with mathematical modelling by observing and interviewing them in the context of a regular, if unique, course on mathematical modelling. The framework illustrates how different sources of authority as well as conceptions and misconceptions of mathematics and mathematics modelling play different roles in the mathematical modelling process. Technology acted both as a tool and as a source of authority in this process.

The studies were conducted at the University of Gothenburg during the fall semester of 1997, the spring semester of 1998, and the fall semester of 1998. A qualitative approach was used in which special attention was focused on a small group of students working together in the laboratory. Data were collected from questionnaires, videotaped interviews, observations, and written documents such as course assignments and examinations. The first study revealed that the students in general favored the use of technology, especially when solving complex mathematical modelling problems. On the other hand, they easily "got lost" and trusted the technology far too much when working on mathematical modelling problems, thereby neglecting a necessary validity check. This trust, in turn, seemed to profoundly disturb their ability to relate mathematical models to reality. The second study, in addition to verifying the findings from the first, indicated that the students had misconceptions associated with their knowledge of mathematics, of technology, and of problem contexts. A major finding of the third study concerned a transformation of authority that occurred after the first few weeks of the course. The students became rather uncritical of the results they got from the computer or graphing calculator despite the fact that in lectures and laboratory sessions they had been urged to be very cautious when employing software to select models. All three studies confirmed the essential role played by the validation part of mathematical modelling when technology is present.

Keywords: Mathematical Modelling, Assessment, Authority, Responsibility, Open-Ended Questions, Technology, Teacher Education.

The dissertation may be downloaded from the University of Georgia in Acrobat Reader format via my home page at http://ma-serv.did.gu.se/matematik/Thomas.htm.

- **Bergqvist, T.** (2001). *To explore and verify in mathematics.* (Doctoral thesis, University of Umeå, Department of Mathematics, 21.) Umeå: Umeå University. University of Umeå, Department of Mathematics. Opponent: Dr. Morten Blomhøj, Roskilde Universitet, Danmark.

No abstract available, but a short summary in form of a press-release (translated from Swedish by Swedish Research Council):

This thesis deals with mathematics teaching in upper secondary school, especially how an investigative way of working is used and can be used. The results indicate that upper secondary school students both can and want to investigate mathematics at a relatively high level of abstraction, but that such activity is quite unusual.

Students employ almost all their school time in training routine skills, something that can lead to neglect of the mathematical content. To a very great extent the students use superficial images from memory and trained, often incomprehensible, procedures for solving problems. Such a strategy proves deficient when the students encounter new problems or when deeper mathematical reasoning is required. Investigation and discussion of mathematical concepts and issues can be a means of giving the students a better understanding of mathematics, and thereby opportunities to treat more genuine problems. This is the first doctoral thesis in mathematics with an orientation toward didactics of mathematics to be presented at Umeå University.

- **Emanuelsson, J.** (2001). *En fråga om frågor : hur lärares frågor i klassrummet gör det möjligt att få reda på elevernas sätt att förstå det som undervisningen behandlar i matematik och naturvetenskap. [A question about questions: how teachers' questions in the classroom enable knowledge about students' ways of understanding what is taught in mathematics and natural science.] (Göteborg Studies in Educational Sciences, 168.)* Göteborg: Universitatis Gothoburgensis. University of Gothenburg, Göteborg Studies in Educational Sciences.

Abstract
This thesis reports results from a study that focuses on how teachers can learn about their students' learning in mathematics and science.

Current perspectives on learning are positioned in terms of the acquisition and participation metaphor. A third metaphor, the constitutive metaphor, is

proposed and elaborated as an alternative for the current study. The theoretical framework draws upon and tries to further inform phenomenography and the "theory of variation".

The empirical material was generated from audio-taped classroom interactions and follow-up interviews. With the aid of concrete examples, teachers were probed on their understandings of selected parts of the interaction observed.

The results are described in terms of variant and invariant aspects of possible learning objects for the teacher. I pay particular attention to what the students possibly focus upon, and how they deal with the focused content in three different zones – the topical, the conceptual and the procedural zone.

The outcome of the study is discussed in relation to teachers' knowledge. Pedagogical content knowing is scrutinised and a complementary perspective where teachers' knowledge is viewed as constituted by different contextualisations of the subject matter is used to illustrate the interdependency of content as a discipline, content as taught in school and content as understood by students.

The result shows that teachers have, relatively speaking, small possibilities of making distinctions within the conceptual zone. In mathematics the topical zone dominates the interaction, in science the procedural. In other words, in mathematics the teachers mainly open for possibilities to learn, if their students remember facts and procedures; in science how they perform presentations and experiments. In both areas possibilities to make distinctions of qualities in how the students understand the content handled are rather small.

The most important finding of the study is: In order to make distinctions in relation to other persons' ways of understanding something, this thing must be kept invariant and acts of knowing must be allowed to vary in relation to the invariant object of knowing.

- **Lithner, J.** (2001). *Undergraduate learning difficulties and mathematical reasoning* (Tekster fra IMFUFA, 399). Roskilde: Roskilde University.
  University of Umeå, Department of Mathematics.
  Opponent: M. Niss (Roskilde).

Abstract

This PhD thesis consists of five parts: (1) A literature survey and an overview of a larger research project with the same title, including background and framework. Then follow the four main papers of the thesis, presenting completed studies within the larger project: (2) A study on four students' task solving difficulties, indicating that the students were more focused on what is familiar and remembered, than on mathematical reasoning and accuracy. (3) An extension of the former study by developing an analytical framework, and focusing on the quality of their reasoning. It was found that the reasoning was more ‘ superficially experience-based’ than mathematically based. (4) A study describing in detail how most textbook exercises may be solved without considering the core mathematics of the textbook, mainly by copying solved examples, and how this may lead to the behaviour above. (5) A study of the ways students conduct their homework that, among other things, confirms that they are restricted to using the superficial procedures found in (4).

- **Möllehed, E.** (2001). *Problemlösning i matematik: en studie av påverkansfaktorer årskurserna 4–9. [Problem solving in mathematics: a study of influential factors in grades 4–9.]* (Studia psychologica et paedagogica. Series altera, 157.) Malmö: Institutionen för pedagogik, Lärarhögskolan.
  Lund University, Department of Educational Research, Malmö.
  Opponent: Ole Björkqvist, Åbo Akademi, Finland.

Abstract
The purpose of this study is to identify the factors influencing pupils when they individually solve problems in mathematics in grades 4–9 in the compulsory school (ages: 10–16 years). The method that has been used is an analysis of the solutions of 25 problems given to each grade. Some of the problems were the same in all grades and about 100 pupils participated in each grade. Through an examination of the errors made by the pupils it has been possible to extract some abilities, which I have called factors, influencing problem solving. The factors have been indirectly described through categories of errors made by the pupils. 16 factors have been identified; 9 factors belong to the cognitive development, 6 to the mathematical learning in school and one to an individual feature of carelessness. As a result of the extracted factors a model of problem solving was built including all factors. Of all errors made by the pupils in all problems the cognitive factors contributed with about 60 %, the mathematical factors with 25 % and the individual factor with 15 %. This was valid in all grades. Comparisons between grades show that there is a great distribution of the pupils’ ability in problem solving. On an average 25 % of all pupils in grades 4 and 5 are able to solve a problem which 25 % of all pupils in grade 9 do not manage to work out. Comparisons between girls and boys have also been made concerning the proportion of correct answers in all problems. In grades 4 and 5 there are small differences but in grades 6–9 the boys have about 20 % more correct answers than the girls.

- **Palm, T.** (2002). *The realism of mathematical school tasks: features and consequences.* (Doctoral thesis, University of Umeå, Department of Mathematics, 24.) Umeå: Umeå Universitet.
  University of Umeå, Department of Mathematics.
  Supervisor: J. Lithner. Opponent: Rudolf Straesser, Luleå Tekniska Universitet.

No abstract available, but a short summary in form of a press-release (translated from Swedish by Swedish Research Council):

A thesis by Torulf Palm shows that a closer connection with reality in mathematics teaching can increase the ability of pupils at the intermediate level to solve mathematical problems.

The thesis includes a study of 161 pupils' work with applied mathematical tasks. The study demonstrates that many pupils in grade 5, just as in other countries around the world, have a strong tendency to ignore their everyday knowledge when solving mathematical problems. They give answers which disagree with the 'real' situations described in the tasks. Among other things the pupils had to answer how long a time it would take for a person to run ten kilometres. They were told that the person ran 100 metres in ten seconds. Most of the pupils assumed that the person would run equally fast all the time, even though most knew that a runner is unable to keep up a 100-metre pace for 10 kilometres. The study implies that a greater faithfulness to reality in the tasks can heighten the pupils' inclination to take account of realistic factors.

The two most common causes of ignoring realistic factors seem to be the pupils' attitude toward mathematics and the use of superficial strategies for solution. Attitudes that influence the pupils' procedure in these situations include, for instance, ideas such as that there is always a single clear answer to a mathematical task. The pupils do not reflect enough on the practical applicability of their solution methods and answers, and are thus unaware that their answers are implausible.

The identified strategies for solution resemble those which, in other investigations by the Department of Mathematics at Umeå University, have been indicated to be a contributing cause of students' difficulties in mathematics at the university level. The results in the thesis suggest that mathematics teaching which at all levels includes a larger proportion of tasks where these solution strategies and ideas about mathematics are not effective would promote the development of the pupils' general knowledge in mathematics. Moreover, the results suggest that this development could be furthered by greater realism in tasks of applied mathematics.

Torulf Palm, together with a research group at Umeå University, has received funds from the Swedish Research Council to study pupils' difficulties in mathematics and to present proposals for how these difficulties can be counteracted.

- **Bentley, P-O.** (2003). *Mathematics Teachers and Their Teaching.* (Göteborg Studies in Educational Sciences, 0436-1121; 191.) Göteborg: Göteborgs Universitet.
  University of Gothenburg, Göteborg Studies in Educational Science.

Abstract
The aim of the present thesis is to analyse and describe the roles, meanings and interaction of a number of policy variables and aspects of the teaching process in mathematics. A survey study based on a simple, random sample of 724 maths teachers in the Swedish secondary school was carried out using a questionnaire based on a pilot study. The questionnaire included ratings on a Likert-scale, multiple-choice questions and had a response rate of 73%. Educational conditions were examined by factor and cluster analyses. Four constellations were found ranging from 'heavy' to 'very light'.

An empirical model was created of seven items and consisted of ten conceptually different teaching approaches. In the different teaching approaches, the means of the policy variables 'number of pupils taught', 'content knowledge', 'pedagogical content knowledge', 'teaching experience' and 'financial resources' do not differ significantly. Aspects which are part of a procedure of the teaching process have characteristic roles and meanings in each one of the teaching approaches and in some cases the means of the aspects differ significantly between the approaches.

The variety of maths teaching made it possible to analyse the relationship between policy variables and aspects of the teaching process by simple regression analyses. Not only strong but also weak relationships were found in the different teaching approaches. The variation of maths teaching is thus constituted not only by the different teaching approaches but also by variations within each approach. There exists a variation in each of the teaching approaches that is dependent on the policy variables.

The simple random sample allows conclusions to be drawn about the population and helps constitute a better foundation for policy decisions that could lead to improved teaching outcomes. In addition, knowledge of the different teaching approaches should be part of teacher training and in-service training. By a wider battery of rating items, it would be possible to obtain a more detailed map of the influence of policy variables on the teaching process.

- **Samuelsson, J.** (2003). Nytt, på nytt sätt? – En studie över datorn som förändringsagent av matematikundervisningens villkor, metoder och resultat i skolår 7–9. [New in a new way? – A study of the computer as agent of change in mathematics teaching's conditions, methods and results in school years 7–9.] Uppsala Universitet: Pedagogiska Institutionen. University of Uppsala, Department of Education. *Supervisor*: U. Riis, Univ. of Uppsala. *Opponent*: S. Lindblad.

Abstract

There are several documents that indicate that computers will change the conditions for mathematics teaching. In this dissertation the author discusses how the computer as a change-agent will influence the conditions, methods and results in mathematics teaching. To that purpose the constructs inspired by activity theory a model of analyses focusing on the relations between these three components. The empirical material is collected through interviews with eighteen teachers in lower secondary school. The author has also participated in all computer-aided lessons given by two teachers during one year. That means 700 possible computer-aided lessons. Teaching of mathematics seems to have such a strong tradition that the computer as a change-agent is relatively weak. The fact is that the computer is assimilated into an old tradition of methods and contents. A great deal of the computer-aided lessons give attention to drilling pupils with different types of drill-program where they can learn mathematical procedures. In some lessons laborative work is pursued with the intention that the pupils should learn mathematical concepts. An important condition for this type of work is the speed of the computer, a condition which contributes to teachers drawing attention to many different forms of mathematical knowledge.

- **Löwing, M.** (2004). *Matematikundervisningens konkreta gestaltning – En studie av kommunikationen lärare – elev och matematiklektionens didaktiska ramar. [The concrete shaping of mathematics teaching – A study of teacher-pupil communication and mathematics classes' didactic framework.]* (Göteborg Studies in Educational Sciences, 208.) Göteborg: Göteborgs Universitet. University of Gothenburg, Göteborg Studies in Educational Sciences. Supervisor: B. Lindström, University of Gothenburg. Opponent: G. Gjone, Universitetet i Oslo.

No abstract available, but a press-release (translated from Swedish by Swedish Research Council):

The teacher and teaching in mathematics

Madeleine Löwing has recently presented a doctoral thesis in didactics of mathematics, entitled *The concrete shaping of mathematics teaching*. It describes what happens during a number of mathematics classes in school years 4–9. The thesis is of interest because it answers several of the questions that have lately been posed about the state of Swedish teaching in mathematics.

Lowing maintains that the teachers she has studied are regarded as capable and well-educated teachers by their colleagues and school administrators. She gets the same impression when sitting in the classroom. The teachers are careful to focus on the pupils, they have good social relations with the pupils, and they generally use modern methods and materials for instruction. The problems raised by Löwing in the thesis are not noticed until one analyses the communication that occurs between the teacher and pupils. This communication has been documented with the help of, for instance, a microphone worn by the teacher.

Löwing first analyses what happens on a macro-level, i.e. how the teacher plans and builds up the instruction. It emerges that the teachers' choices of instruction strategies are not always the best. They often have an ambition of using several modern methods simultaneously. However, these choices often prove to counteract each other. For example, the pupils are placed in groups so that they can discuss mathematics and give mutual help. But the groups' composition has been made by the pupils themselves, and thus for social reasons, not on the basis of the instruction's goals. Since the pupils work at their own pace, guided by educational aids, there is such a large spread in the group after only a couple of days that most of the pupils are working with quite different tasks and have little interest in helping each other. Instead many private conversations arise, and often continue during most of the class time.

When Löwing next analyses the classes on a micro-level, i.e. the content of what the teacher and pupils discuss, a number of problems with the mathematics teaching are seen clearly. One such problem is that the teachers in general have "abdicated" as work leaders and instructors, and turned over the responsibility to an educational aid. This is due partly to the teacher feeling a pressure to individualise. The teachers think that, by working at their own pace, all the pupils get the time they need to solve their tasks. Actually the effect is the opposite. As the pupils cannot discuss mathematics, they do not build up a usable language. Hence they do not master the expressions and methods which are used in the book, so they find it hard to understand the book's instructions. Since many pupils have the same difficulty with understanding the text, and need help, there are long waiting times and many pupils are simply forgotten about. This leads to more conversation and disturbance than mathematics.

Another interesting observation is that the teacher and pupils often talk at cross-purposes, for two reasons. First, the teacher is frequently too stressed to take time for listening to the pupils' actual problems, and therefore explains not these problems but what the teacher thinks their problems are. Secondly, most of the teachers have no control over the pupils' previous knowledge. The pupils consequently have difficulty in keeping up. By asking clever leading questions, the teacher usually manages to guide the pupil to a correct answer – yet without the pupil having understood. As a result, the pupil also has trouble solving the next task.

A further observation is that the teacher and the authors of educational aids often have different views of which method is suitable. The consequence here is that a pupil who has not understood the book's explanation gets a completely different, contradictory explanation from the teacher, and becomes even more confused.

List of licentiates

- **Engström, L.** (1999). *Teaching based on a constructive perspective supported by a computer program.* London: South Bank University.
  South Bank University, London.
  Supervisors: P. Winbourne, S. Lerman.

Abstract
This paper examines a computer program in mathematics that helps the teacher to base teaching on a constructivist perspective. It analyses one pilot project involving this program in a secondary school in 1998.

The project shows the experiences of using a dynamic geometry program called CABRI. The result focuses mainly on how conceptions in geometry can be made understandable and fortified.

The following are the main questions in my project:

Can this program help the students to understand basic concepts in Euclidean geometry? How might teachers best employ computer assisted learning? Does the computer program help the teachers to teach from a constructivist perspective?

This project is based on the belief that there are computer programs that can help the teacher to teach from a constructivist perspective. I concentrate on one program, CABRI, which deals with Euclidean geometry. This program is not yet well known in Sweden. I describe it and give some examples. Geometry, which from the beginning was a subject of its own, exclusively for boys, is now a minor

part of the mathematics curriculum. I will give the history of geometry in the Swedish curriculum as a background as to why I chose geometry. The project with the CABRI program has been carried out in classes of 13- and 15-year-old students. This project will be used as an illustration of the possibilities and will give information on my reflections. I report on interviews with some teachers and a half-day of work I had with students who have used CABRI. The finding was that concepts in mathematics become easy to understand and remember. My conclusion is that when students have a chance to formulate their own facts, found from experimentation, they learn and find the work interesting.

- **Bjerneby Häll, M.** (2002). *Varför undervisning i matematik? Argument för matematik i grundskolan – i läroplaner, läroplansdebatt och hos blivande lärare. [Why teach mathematics? Arguments for mathematics in compulsory school – in curricula, in debate on curricula, and among student teachers.]* Linköping: Linköping University.
  Linköping University, Department of Behavioural Science.

Abstract
The aim of this study is to investigate, describe and analyse the justification and the goals of mathematics education in compulsory school in Sweden. The focus here is on student teachers and their conceptions of why mathematics should be taught in school.

A qualitative approach has been used and data (short texts) have been collected from five cohorts of student teachers during their first and second year of education respectively. The data were interpreted on a conceptual level, in the descriptions of arguments for mathematics. A study of curriculum texts and debates has served the purpose as background and context related to the arguments presented by the student teachers.

The outcome of the analysis of the student teachers' texts resulted in ten basic arguments for school mathematics. The main arguments are of either functional or formal character.

- **Bremler, N.** (2003). *Matteboken som redskap och aktör – en studie av hur derivata introduceras i svenska läroböcker 1967–2002. [The mathematics book as tool and actor – a study of how derivatives are introduced in Swedish textbooks, 1967–2002.]* Stockholm: Lärarhögskolan Stockholm.
  Stockholm University, Institutionen för ULK.
  Supervisor: S. Selander, Stockholm Institute of Education.
  Opponent: C. Bergsten, Linköping University.

Abstract

The basis of the present work is the idea that textbooks in mathematics are essential for students as well as for teachers. Mathematics textbooks are ubiquitous elements in the learning and teaching process: when teachers teach and students learn in the classroom, when students do their homework, when parents try to help their children understand mathematics and when teachers plan their lessons. The mathematics textbook can be seen as an important tool but it can also be seen as a main figure in mathematical education. The latter is true because the textbook speaks to its readers: instructing them, testing them, trying to show them what mathematics is all about and, in a sense, defining mathematics as a school subject. A textbook in mathematics is also important in the construction of the discourse of school mathematics, with its special language, attitudes and "rules". Some of these "rules" are known as the didactical contract.

The aim of this study is to investigate, analyse and discuss what the readers – students, teachers, parents and others – of Swedish upper secondary school mathematics textbooks are being offered. The study focuses on the introduction of the derivative concept in Swedish textbooks published in 1967–2002. Several important areas have been included in the study. It shows that the history of mathematics has become more common in newer textbooks but it is often presented with short texts dealing with persons and seldom with mathematics. An outspoken purpose, an answer to the question 'Why learn mathematics?', is rare in the textbooks and the ones presented often designate that mathematics is needed so that the students will be able to cope with higher levels of mathematics. It would seem fairly obvious to make use of dynamic images or series of images in order to explain the geometric interpretation of the derivative, but these are only used in about half of the books. The proof of the differentiation formula often appeared in older books but has totally disappeared today. Most exercises in the textbooks can be solved by copying and sometimes slightly modifying the solved examples. The most frequent type of voice speaking from the textbooks is a formal and all-knowing monologue. There are exceptions, e.g. fictive student voices and humorous voices. Images, informative ones but decorative ones in particular, have become more common in the textbooks. It is uncertain though, if the latter make it easier for students to learn mathematics. "Real life" tasks/exercises that are realistic regarding situation, formulas and openness are rare. However, it is not certain that true realism always is possible and/or desirable in school mathematics. When it comes to gender issues, only about a third of all the persons appearing in the books are female. Gender-role stereotyping is not common but it does exist, even in more recently published textbooks. Finally, an "external" area of the study shows that an overwhelming

majority of the authors are men and that a small group of "veteran authors" dominate the writing of mathematics textbooks.

Since the mathematics textbook is so important in school mathematics, it can be said to form the discourse of school mathematics. Thus, the qualities of the textbooks found in this study can also be seen as characteristics of the discourse of school mathematics.

- **Johansson, M.** (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum.* Luleå: Luleå Tekniska Universitet. Luleå University, Department of Mathematics.
  Supervisor: L-E Persson. Co-supervisors: B. Grevholm and R. Strässer.
  Opponent: Ole Björkqvist, Åbo Akademi, Finland.

Abstract
Textbooks are a most important feature of teaching mathematics in the classroom, in Sweden as well as in many other countries. For teachers and students, the textbooks often determine what is school mathematics and also what is mathematics. Previous studies on textbooks and the use of textbooks in teaching and learning mathematics raise important questions about textbooks as representations of the curriculum. One important question concerns their role as a link between curriculum and activities in classrooms. In this thesis, some international investigations in this connection are reviewed and analyzed. Moreover, in order to illustrate the role of textbooks as the potentially implemented curriculum, a content analysis of a textbook series is conducted. The development of the textbook series, a commonly used schoolbook in Sweden, is portrayed in the light of the curriculum development. Some findings from the analysis of the textbooks show that the objectives of mathematics, formulated in the national curriculum, are only partially realized.

- **Nilsson, P.** (2003). *Elevers förståelse av en slumpsituation – En fallstudie av hur elever i årskurs 7 tolkar och hanterar aspekter av sannolikhet aktualiserade i ett tärningsspel. [Pupils' understanding of a chance situation – a case study of how pupils in grade 7 interpret and handle aspects of probability actualised in a dice game.]* Växjö: Växjö universitet.

Abstract *(translated from Swedish by Swedish Research Council)*
This study has investigated grade 7 pupils' way of handling aspects of probability actualised in an experimental situation. Eight pupils divided into four groups of two worked with optimising strategies for increasing their chances of winning

in a dice game, based on the sum of two dice. The situation investigated was divided into four rounds, with a new set-up of dice in each round. The dice were designed to actualise different aspects of probability and, at the same time, the pupils were given an opportunity to encounter small changes in mathematical structure between different situations.

How pupils understand and develop concepts is viewed in the situation from the standpoint of how their understanding varies with their interpretations of the situation in which the concepts occur. Such a significance-generating process is described by the study in terms of differentiation and contextualisation. Context is then given by pupils' personal constructions, and the concepts that are actualised in a learning situation fall within the framework of the *conceptual* part of the context – in the same way as ideas about the learning situation belong to the *situational*, and the perception of norms and values to the *cultural*, context (Halldén, 1999).

Starting from this learning perspective, the study aims to explain pupils' manner of understanding and handling aspects of probability – to describe pupils' different ways of contextualising tasks that actualise such aspects, given the conceptual and situational/cultural resources at their disposal.

Empirical data have been analysed with intentional analysis, a method based on regarding the pupils' actions as intentional. By attributing intentions to the pupils we can render their actions plausible and thereby consolidate the structure of the interpretation work.

The analysis shows the importance of setting pupils' conceptual repertoire in relation to the way they process information and place it at their disposal. The pupils' perception of aspects of probability must be put in relation to their manner of creating meaning in a task situation.

- **Ryve, A.** (2003). *Concept mapping in linear algebra.* Västerås: Mälardalens Högskola.
  Mälardalen University.
  Supervisors: K. Eriksson, I. Wistedt.

Abstract *(translated from Swedish by Swedish Research Council)*
This licentiate thesis contributes to strengthening research on group work in mathematics, partly by studying the communication between engineering students in groups constructing conceptual maps, and partly by evaluating and developing the methodological frameworks that are used in such studies.

Four groups of engineering students were video-filmed while constructing conceptual maps in linear algebra. The purpose of the exercise was that the

students should think about the relations between different mathematical concepts they had encountered in the course. By analysing how the groups worked, one can try to answer key questions regarding how well the exercise fulfils its purpose. Do the students communicate in an effective manner? Is their communication mathematically productive?

To answer the questions I used the following methodology. First a study was made of whether the students used the expressed concepts in a similar way. Next came an analysis of how the students communicated with each other and whether they communicated about mathematics. In terms of these criteria, I found that the communication between the students could be considered mathematically productive.

One can, however, question whether the above method of analysis suffices to answer the questions fully. Therefore I show how the method can be strengthened with two methodological tools: a so-called *intentional analysis* that may give better answers to whether the students communicate with each other, and a *differentiation between mathematical expressions and arguments* that may provide a more subtle picture of the communication.

Based on the study, I propose that future research concerning the concept of "mathematically productive communication" should define the concept so that criteria can be derived from the definition. Further, I emphasise the importance of distinguishing between types of mathematical communication and of investigating whether the students really communicate with each other.

- **Taflin, E.** (2003). *Problemlösning och analys av rika matematiska problem. [Problem solving and analysis of rich mathematical problems.]* Umeå: Umeå Universitet.
  Umeå University, Department of Mathematics.

Abstract

In this dissertation I will define and explore mathematical problem solving. I will also try to find out and formulate criteria for problems that should be used in classrooms for education. The criteria shall also define some goals for education in mathematics. Problems that fulfil the criteria will be called rich problems. These rich problems shall be used when pupils learn specific mathematical concepts and processes. The first part of the dissertation is a literature study, and I will end the study by identifying seven criteria of rich problems. The second part of the dissertation is an analysis of three problems to find out in what way they are rich. The analysis consists of a theoretical part where I show the mathematical ideas, possible strategies, and representations of the problems. After the theoreti-

cal part I will show examples of pupils' solutions. I have found out that pupils with different mathematical backgrounds can work with the same mathematical problems and solve them with different mathematical ideas and strategies.

- **Juter, K.** (2004). *Learning Limits of Functions, university students' development during a basic course in mathematics.* Luleå: Luleå Tekniska Universitet. Luleå University, Department of Mathematics.
  Supervisors: L-E. Persson, B. Grevholm. Opponent: J. Lithner, Umeå Univ.

Abstract
The present thesis includes four articles and an introduction to them and the theories that frame them. The articles describe a study about students' learning limits of functions at a university and the results from it. The main question is: How do students deal with the concept of a limit of a function at the basic university level in Sweden? In an attempt to answer this big and wide question, a set of narrower questions are considered: What are the results of students' creations of mental representations of limits of functions? Do the representations change during the time of the study? What changes if anything? Do high achievers' representations differ from low achievers' representations of limits of functions? How do students solve problems with limits?

How do they explain their solutions? What effects can attitudes to mathematics and actions in the classroom have on the learning of limits of functions?

Different methods were used in order to seek answers to the questions. They were questionnaires at different times during the course, field observations and interviews. The study was divided in two parts with a pilot study with 148 students and a main study with 112 students. The limit concept is an important part of the foundations of mathematical analysis and if the students do not understand clearly what it is about, they can have problems when they are dealing with concepts such as continuity and derivatives. No such study on limits has previously been done in Sweden so it is compared mostly to foreign results. It is important that people who work with mathematics education at university level are aware of the situation the students are in.

Appendix B

Anna Friberg & Rudolf Strässer

Swedish institutions

(Mathematics and Pedagogy/Education) with activities in Didactics of Mathematics according to their websites

According to Högskoleverket, at http://katalogen.sunet.se/kat/education/universities, a catalogue of all universities and institutes in Sweden can be found. Starting from this site and using the Internet as basic source, we identify persons, groups and institutions with activities related to research into Didactics of Mathematics at the different universities or institutes. The presentation only includes universities and university colleges which have research and education in Didactics of Mathematics. Some of the websites are written only in Swedish. The information is preferably presented in English, but may be available only in Swedish.

University College of Borås

Homepage: http://www.hb.se

At the website http://www.hb.se/ped/forskning/forskn.htm, no research in the area of didactics of mathematics is indicated. All that was found was an "ämnesdidaktisk kollegium". Here, teachers can discuss different areas in didactics that interest them.

List of people working in the area of Didactics of Mathematics:
Susanne Björkdahl Ordell, Gunnar Nilsson, Elisabeth Persson.
No **publications** in the area of mathematics education were found.

University of Gothenburg/ Chalmers University of Technology

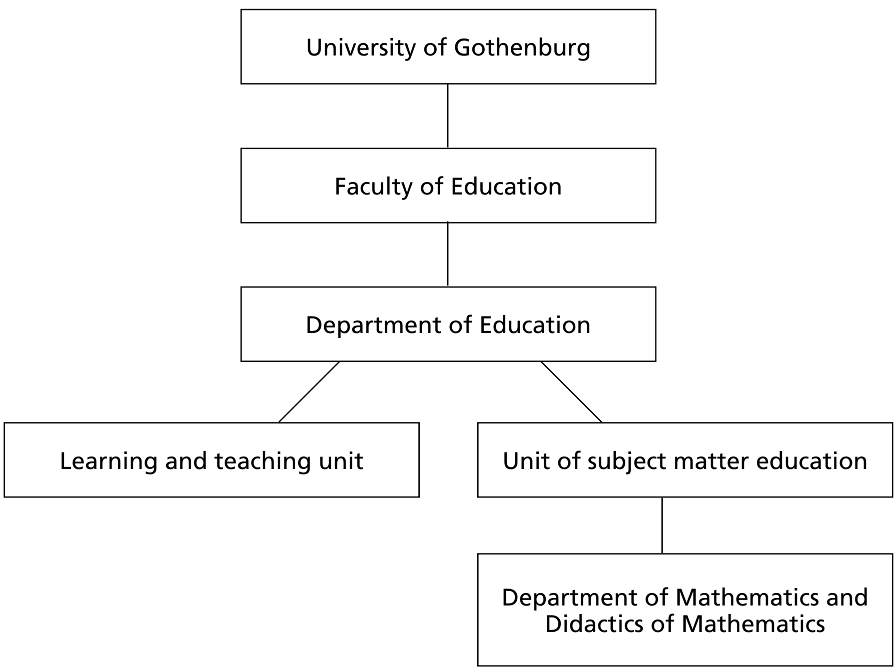
Homepage: http://www.gu.se / http://www.chalmers.se/

Of interest here is the Faculty of Education, which is divided into four different departments. Research in didactics is found in the Department of Education (IPD) with homepage: http://www.ped.gu.se/ipd/. Here we find two units: The learning and teaching unit and the Unit for subject matter education. At the latter there is a link to the Department of Mathematics and Didactics of Mathematics (see drawing top of next page!).

The homepage of the Department of Mathematics and Didactics of Mathematics is located at http://ma-serv.did.gu.se/.

List of people working in the area of Didactics of Mathematics:

Per-Olof Bentley – Universitetsadjunkt, Marie Fredriksson – Universitetsadjunkt, Åse Hallberg Hansson – no information found, Mikael Holmquist – Universitetslektor, Johan Häggström – no information found, Kaj Jönsson – no information found, Lisbeth Lindberg – Universitetslektor, Katarina Lindgren – Universitetsadjunkt, Thomas Lingefjärd – Universitetslektor, Madeleine Löwing – Universitetsadjunkt, Ilse Rossi – Universitetsadjunkt.



One should additionally mention the international 'KULT (Svensk skolkultur: Klassrumspraktik i komparativ belysning)' project, which looks into mathematics teaching (see http://extranet.edfac.unimelb.edu.au/DSME/lps/subresearch.shtml#sweden) and which is based in Göteborg at the Department of Education (team leader in Göteborg: Ference Marton; additional members in Göteborg: Jonas Emanuelsson, Bengt Johansson, Johan Haggström, Johan Liljestrand, Sverker Lindblad, Livia Norström, Ulla Runesson).

Publications related to these institutions:

- Dissertations at the Learning and teaching unit in "Göteborg Studies in Educational Sciences" (ISSN 0436-1121); homepage: http://www.ped.gu.se/forsk/rapporter/avhandlingar.html
- Reports about mathematics at the Unit for subject matter education; homepage: http://ma-serv.did.gu.se/matematik/rapport.htm
- IPD- reports from 1999- (Swedish and English, ISSN 1404-062X); homepage: http://www.ped.gu.se/forsk/rapporter/ipd-rapporter.html
  The School of Mathematical Sciences is connected with both Chalmers and the University of Gothenburg.

In addition to this, another institution has to be mentioned in Göteborg:

Nationellt Centrum för Matematikutbildning(NCM)

Homepage: Http://ncm.gu.se
We cite directly from NCM's English webpage:
"The Swedish government decided to establish a national resource center for mathematics education at Göteborg University with effect from January 1st, 1999. The enterprise will be carried out in collaboration with Chalmers University of Technology. NCM will co-ordinate, support, develop and implement the contributions, which promote Swedish mathematics education from pre-school to university college."
At present (June 2004), NCM's homepage lists 21 persons working at the institution, who include administrative personnel. Major activities of NCM are the editing of two journals ("NOMAD" and "Nämnaren") and the development of the library and documentation of publications within the domain of Didactics of Mathematics. A comprehensive description of NCM's activities is found at http://ncm.gu.se/index.php?name=verksamhet-startsida.

University of Jönköping

Homepage: http://www.hlk.hj.se/
List of people working in the area of Didactics of Mathematics:
Ann Ahlberg – professor.
No **publications** in the area of mathematics education were found.
As a special activity linked to Jönköping, the "SUM" network should be mentioned. "SUM" stands for Svenskt nätverk för särskilda utbildningsbehov i matematik. Co-ordinator is Professor Ann Ahlberg, Högskolan för lärande och kommunikation, Jönköping. Secretary is doctoral candidate Elsa Foisack, Lärarutbildningen, Malmö Högskola.
Homepage of SUM: http://www.hlk.hj.se/doc/405
Publications:
Konferensrapport Demokrati och delaktighet, Örebro 2003.

University of Kalmar

Homepage: http://www.hik.se/
List of people working in the area of Didactics of Mathematics:
Valeri Marenitch – professor.
At the website http://www.hik.se/forskning/, we find that research is being done in Mathematics education at this university. There is a project "To Win Math" (Att "vinna" matematiken), which is supported by Vetenskapsrådet. No publications were found on the website.
No **publications** in the area of mathematics education were found.

University of Karlstad

Homepage: http://www.kau.se/
List of people working in the area of Didactics of Mathematics:
At this University there is one Ph.D, Anna Löthman, who is doing research in pedagogy with focus on didactics of mathematics.
Publications of Anna Löthmann can be found at http://www.kau.se/forskning/forskdb/index.lasso?to_do=show_forskare&ID=1163.

Kristianstad University College

Homepage: http://www.hkr.se/
At Kristianstad University research in science and mathematics education is carried on at the Department of Mathematics and Science (MNA). The research group carries on various projects about learning at different ages. The group is called LISMA, which stands for Learning in Science and Mathematics (homepage: http://www.mna.hkr.se/lisma/index-en.htm). The leader and co-ordinator of the group is Professor Gustav Helldén. The group was established in 1994, and has 18 members of staff who are involved in different research projects:
List of people working in the LISMA group:
Olle Eskilsson, *Barbro Grevholm, Lena Hansson, *Örjan Hansson, Gustav Helldén, *Ingemar Holgersson, Inger Holmberg, Bengt Ingvarsson, Kristina Johansson-Tell, *Kristina Juter, Britt Lindahl, *Ann-Charlotte Lindner, *Lena Löfgren, Ola Magntorn, Pernilla Nilsson, Constanta Olteanu, *Christel Persson, Andreas Redfors, Maria Rosberg, Susanne Thulin (people marked with a * teach Mathematics education).
Since 1999, many publications have been produced in Didactics of Mathematics by this group (see http://www.mna.hkr.se/lisma/publikationer.htm).

University of Linköping

Homepage: http://www.liu.se/
This University is connected with the "Forskarskolan med ämnesdidaktisk inriktning". Read more about the latter at the end of this report.
List of people working in the area of Didactics of Mathematics:
Christer Bergsten, Maria Bjerneby Häll, Magnus Österholm.
Research presented on the Internet and connected with Didactics of Mathematics is: Maria Bjerneby Häll: Varför matematik? (on-going project, published: licentiate in 2002). Magnus Österholm: Matematiska texter som grund för lärande i matematik, med fokus på läsprocessen (on-going project, published: konferensbidrag/rapport in 2003) Christer Bergsten: Gränsvärdesbegreppet i grundläggande högskolematematik (planned project).
Publications have been produced by all three persons listed above.

University of Luleå

Homepage: http://www.luth.se
At this university the research in mathematics education is carried out at the Department of Mathematics. The research group in Didactics of Mathematics ("Mathematics and Learning in Luleå – MaLiL" has a homepage at http://www.sm.luth.se/math/research/malil/index.html.
List of people working in the area of Didactics of Mathematics:
Anna Brändström, Barbro Grevholm, Teresia Jacobsson-Åhl, Monica Johansson, Anna Klisinska, Lars-Erik Persson, Anna Sierpinska, Rudolf Strässer, Christina Sundqvist.
Publications have been produced by all persons listed above.

Lund Institute of Technology

No research is being done in the area of Didactics of Mathematics in Lund. Gerd Brandell, Associate Professor at Lund Institute of Technology, is the coordinator for the research school "Forskarskolan i matematik med ämnesdidaktisk inriktning" supported by Riksbankens Jubileumsfond and Vetenskapsrådet (see end of this appendix).

Malmö University

Homepage: http://www.mah.se/
We were unable to find a specific division of didactics of mathematics. According to our information, the area of Didactics of Mathematics is "under construction" at Malmö.
List of people working in the area of Didactics of Mathematics at the School of Teacher Education in Malmö: Elsa Foisack, Olof Magne, Ebbe Möllehed.
Individual **publications** from the three persons mentioned above can be found.

Mälardalen University College

Homepage: http://www.mdh.se/
Information on research in Didactics of Mathematics at Mälardalen can be found at the homepage of the RJ-forskarskolan (research school): http://www.msi.vxu.se/Forskarskolan/. Didactical research at Mälardalen is formally linked to Stockholm University.
List of people working in the area of Didactics of Mathematics:
Kimmo Eriksson – professor in discrete mathematics, Hillevi Gavel – college lecturer, Andreas Ryve – doctoral candidate in the research school.
Andreas Ryve has published in the area of Didactics of Mathematics.

Royal Institute of Technology

Homepage: http://www.kth.se
Information on research in Didactics of Mathematics at the Royal Institute of Technology (KTH) can be found at the homepage of the RJ-forskarskolan (research school): http://www.msi.vxu.se/Forskarskolan/. Didactical research at KTH is done within the "Knowledge Management Research" group.
List of people working in the area of Didactics of Mathematics:
Ambjörn Naeve – professor, Mikael Nilsson – doctoral candidate in the research school. Both persons have published in the area of Didactics of Mathematics.

Stockholm Institute of Education (Lärarhögskolan Stockholm)

Homepage: http://www.lhs.se
There are two projects to be mentioned at Lärarhögskolan Stockholm:
Under the supervision of Staffan Selander, Niklas Bremler worked on mathematics textbooks (see http://www.lhs.se/forskning/projekt/projekt_132.htm). He also produced a licentiate thesis on this topic.
An interdisciplinary project with a strong mathematical component is also run at Lärarhögskolan Stockholm: the PRIM group (**PR**ov **I** **M**atematik) was created in 1984 to develop the national assessment in mathematics (see http://www.lhs.se/resunits/prim/).
(Incomplete) **list of people** working in the area of Didactics of Mathematics:

Niklas Bremler, Jonas Ingemarsson – project assistant, Katarina Kjellström – senior lecturer, Bengt-Olov Ljung – professor emeritus, Astrid Petersson – professor, project leader of PRIM group, Staffan Selander – professor.
Publications: See the work of the PRIM group at http://www.lhs.se/resunits/prim/ in the area of mathematics education.
The work on the textbook is reported in Bremler's licentiate thesis.

Stockholm University

Homepage: http://www.su.se/
List of people working in the area of Didactics of Mathematics:
Agneta Andersson – teacher and starting research, Clas Löfwall – professor in mathematics, Kirsti Nordström – Ph.D. student of the RJ research school, Inger Wistedt – professor in education.
Individual **publications** on Didactics of Mathematics can be found. No publication series.

Umeå University

Homepage: http://www.umu.se, more specific: http://www.math.umu.se/Didaktik/index.html.
List of people working in the area of Didactics of Mathematics:
Tomas Bergqvist, Ph.D. in mathematics with orientation toward didactics of mathematics, Johan Lithner – professor in didactics of mathematics, Torulf Palm, Ph.D. in mathematics with orientation toward didactics of mathematics, Nina Rudälv – Ph.D. in mathematics, Hans Wallin – professor in mathematics.
Ph.D. students: *Jesper Boesen, Olof Johansson, Eva-Stina Källgården, *Peder Långström, Eva Taflin, *Lovisa Ulfsdotter (students of the RJ research school are marked by *).
Research reports on mathematics education can be found at the homepage:
http://abel.math.umu.se/Didactics/MathsEdRep.html
Dissertations can be found at: http://abel.math.umu.se/Didactics/MathsEdDiss.html
For Umeå University, one should additionally mention the research done on "National Tests in Mathematics" at the Department of Educational Measurement (see http://www.umu.se/edmeas/np/). Although formally linked to the Faculty of Social Sciences, assessment of mathematical learning at upper second-

ary level ('gymnasieskola') is a major research focus – with Peter Nyström and Torulf Palm to be mentioned as known researchers in Didactics of Mathematics. In June 2004, the homepage of the project mentioned 20 persons as working on the project. Publications can be found at http://www.umu.se/edmeas/publikationer/.

Uppsala University

Homepage: http://www.uu.se/
At Uppsala, research in Didactics of Mathematics takes place in different institutions. Besides the Department of Mathematics (see below), the teacher training institute ('Institutionen för lärarutbildning') has a research project on concept formation in primary school. The Department of Pedagogy takes part in the international 'KULT (Svensk skolkultur: Klassrumspraktik i komparativ belysning)'project, which looks into mathematics teaching (see http://extranet.edfac.unimelb.edu.au/DSME/lps/subresearch.shtml#sweden and http://www.ped.uu.se/research/projekt.aspx?action=visa&id=120). Team leader in Uppsala: Fritjof Sahlström; researchers: Josefin Häggblom, Johan Liljestrand, Sverker Lindblad. For this information see the "Program för forskning och forskarutbildning vid Utbildningsvetenskapliga fakultetsnämnden 2002–2004" at http://www.utbildningsvetenskap.uu.se/organisation/FOU-program.pdf).
At the Department of Mathematics, Uppsala University, three students are connected with the RJ research school. Kerstin Ekstig is the project leader for Grundutbildningsrådets projekt "Improved understanding of mathematics through computer based problem solving".
List of people working in Didactics of Mathematics at the Department of Mathematics: Gunnar Berg – lecturer, Kerstin Ekstig – associate professor, Sten Kaijser – professor in mathematics, Staffan Rodhe – lecturer, Anders Vretblad – lecturer, Anders Öberg – lecturer. Kajsa Bråting, Johanna Pejlare, Johan Prytz are Ph.D. students of the RJ research school.
No publication series in the area of mathematics education were found. Mathematical publications from Uppsala in general can be found at http://www.math.uu.se/research/.

Växjö University

Homepage: http://www.msi.vxu.se/forskn/forskarskola_matdid.html
List of people working in the area of Didactics of Mathematics:
Ph.D. students: *Torbjörn Fransson, Reza Hatami (Blekinge tekniska högskola), *Per Nilsson, Erika Stadler (Växjö universitets Lärarutbildningsnämnd). (Students of the RJ research school are marked by *.)
Supervision is done by Lennart Hellström – lecturer, Håkan Sollervall – lecturer, Anders Tengstrand – lecturer, and Inger Wistedt – professor in pedagogics.
No **publications** in the area of mathematics education – apart from Per Nilsson's licentiate thesis – were found.

National Research School in Didactics of Mathematics (Nationell forskarskola i matematik med ämnesdidaktisk inriktning)

Homepage: http://www.msi.vxu.se/Forskarskolan/
In autumn 2001 a national research school in Didactics of Mathematics ('nationell forskarskola i matematik med ämnesdidaktisk inriktning') was started. It was and is financed by Riksbankens Jubileumsfond and Vetenskapsrådet.
The homepage of the research school describes the background as follows (quotation from the website, available only in Swedish) (translated from Swedish by Swedish Research Council):
"When the Bank of Sweden Tercentenary Foundation (RJ) decided to allocate funds for a graduate school in mathematics with orientation toward didactics in March 2000, it was also resolved that members of the Swedish Committee for Education in Mathematics (SKM) should be responsible for planning and conducting the graduate education. The SKM members in their new function were termed the executive group for the graduate school. Go to the link 'executive group/presentations' in the menu for more information about the members in the executive group!
The ultimate aim of this commitment by the Foundation was to contribute to giving the subject of mathematics an improved position in schools and universities. Graduate education with the proposed orientation existed at only a couple of places in the country. In order to anchor the plans for a graduate school and to initiate development work at other universities and colleges, RJ supported a dialogue with the relevant research directors at these institutions.

Therefore, RJ invited universities and colleges to a first meeting with representatives from RJ and participants in the graduate school's newly appointed executive group. The meeting was held already in April 2000 and was strongly attended. It provided room for discussions, viewpoints and proposals, which naturally were valuable in the executive group's further planning work. The work continued throughout 2000, and a memorandum for discussion was sent out on referral to departments of mathematics, pedagogy and teacher education in mathematics and natural science. After the referral period, a hearing was arranged by invitation from RJ in Stockholm in September, and it too had many participants. At the meeting in October, RJ's board of directors was able to decide on the graduate school's organisation in accordance with the executive group's proposals, which were based largely upon the viewpoints that arose during the consultation. A preparatory conference/course with prospective supervisors in Oxford took place during December 2000.

RJ had previously appointed a coordinator for the graduate school, with employment at Luleå University of Technology where the administration was placed.

In January 2001, the formal invitation to collaborate in the graduate school was sent to universities and colleges. This meant, in short, that the departments which had possibilities of offering one or more doctoral candidates a good environment for graduate education in mathematics with orientation toward didactics could expect support from the graduate school, for example in the form of 80% of salaries for the doctoral candidate(s) during a five-year period, support for competence development of supervisors, grants for the department's costs and financing of doctoral candidates' travel and lodgings.

Announcement of doctoral candidate posts was made in February 2001 and, after assessing and recruiting the applicants, interested departments could send their applications during April that year to the executive group for further judgement. Decisions on which departments would receive permission to collaborate, and which doctoral candidates would finally be accepted for the graduate education, were taken by the executive group during May 2001."

At present, the following universities and university colleges take part in the RJ research school: University of Gothenburg, Kristianstad University College, University of Linköping, Luleå University of Technology, Mälardalen University College, Royal Institute of Technology Stockholm, Stockholm University, Umeå University, Uppsala University, Växjö University.

Information about the research being done at the different universities can be found at the homepage of the research school.

For information, we add some other pertinent links related to mathematics education in Sweden:
LMNT, Riksföreningen för lärarna i matematik, naturvetenskap och teknik, http://www.lhs.se/lmnt/
SMaL, Sveriges Matematiklärarförening, http://www.smal-matte.com/
SKM, Svenska Kommittén för Matematikutbildning, http://www.sm.luth.se/~gerd/skm/index.html
Svenska matematikersamfundet, http://users.du.se/~fmi/sms/
Svenska Nationalkommitten för Matematik, http://www.math.uu.se/%7Ekiselman/national.html