



Vetenskapsrådet

EVALUATION OF SWEDISH RESEARCH IN MATHEMATICS



Funded by the Swedish Research Council or
the Swedish Foundation for Strategic Research
during the years 2002–2006

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PREFACE

In 2009, the Swedish Research Council together with the Swedish Foundation for Strategic Research initiated an evaluation of Swedish research in mathematics. A previous evaluation of Swedish research in mathematics was last made in 1995¹. Only grants awarded by the funding agencies (the Swedish Research Council and the Swedish Foundation for Strategic Research) between 2002 and 2006 within the area of mathematics were evaluated. The selected time-span would allow for projects, typically of three years' duration, to be finished or nearing the end of their funding period. The Institute Mittag-Leffler² was also included in the evaluation, with the aim of clarifying its importance for Swedish mathematics.

Administration of the evaluation was carried out by Andreas Augustsson, Eva Mineur and Emma Olsson from the Swedish Research Council and Olof Lindgren from the Swedish Foundation for Strategic Research. Further administrative support was provided by Malin Nilsson and Bo Sandberg from the Swedish Research Council.

On behalf of the two funding bodies we hereby express our deepest gratitude to the participating researchers and to the Expert Committee for devoting their time and expertise to this important task. The findings and recommendations are greatly appreciated, and will provide important guidance for future initiatives in the area of mathematics.



Sven Stafström
Swedish Research Council



Lars Rask
Swedish Foundation for Strategic Research

¹ The 1995 evaluation was carried out by the Swedish Natural Science Research Council (which, since 2001, is a part of the Scientific Council for Natural and Engineering Sciences at the Swedish Research Council)

² The Institute Mittag-Leffler is a Nordic institute for research in mathematics.

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EXECUTIVE SUMMARY

The committee has been invited to evaluate Swedish mathematics funded by the Swedish Research Council and the Swedish Foundation for Strategic Research during the years 2002-2006. The results are presented in this report.

The committee finds that Swedish mathematics is strong in many areas; including mathematical analysis with its long historical tradition, an area that for many years was the dominating field of research. More recently there has been increased activity also in the areas of algebra and geometry, with interesting applications to physics. Applied mathematics has become more prominent over latter years, and the committee judges that great progress has been made. Numerical analysis has developed into a very strong field, but the subject is often considered to be part of computer science rather than of mathematics, something the committee believes lead to missed collaborations. Some areas of weakness, such as statistics, where improvements are urgently needed, were also noted. The committee furthermore recommends a build-up of competence in areas related to biology.

The committee finds that the working conditions of mathematicians and of university researchers in general are very difficult in Sweden. The system where funding from the Swedish Research Council is used for reducing teaching loads is considered to be very ineffective. Research time should be guaranteed by the universities, and researchers should not constantly need to apply for grants purely to ensure time for research. The committee also noticed that there seems to be a serious issue with retaining the best young researchers in Sweden.

The committee also observed that the effective amount of funding for mathematics within the Swedish Research Council is very low compared with many other European countries and the USA. This is in part because the demand for funding to reduce the high teaching loads absorbs funds that might otherwise be used for more creative purposes. While it is difficult to make direct comparisons of Swedish funding for mathematics with that in other countries, the committee still felt that the overall level of funding was low for the quality and quantity of the research programs deserving of support.

The committee was very concerned about the situation of the Institute Mittag-Leffler. There is a great danger of losing this national treasure, and the committee strongly urges that addressing the short-term financial shortfall and the long-term financial stability of the Institute be given the highest priority.

The committee was impressed by the range of activities aimed at high school teachers and students. This not only increases the knowledge of mathematics and science in general, but can also encourage students to choose mathematics and science as their future career. In contrast, a weakness was noted in the efforts made by the mathematical community to spread the message about the importance of mathematics in our modern world to the general public. It is encouraged that more Swedish mathematicians communicate their work and findings to the general public, politicians and the press.

Overall the committee concludes that Sweden could make use of, and afford, much more mathematical research than is currently funded. Mathematics is instrumental for the development of new technologies, and investing in mathematics is indirectly investing in the future of Sweden.

1 INTRODUCTION

External funding of research in mathematics derives mainly from the Swedish Research Council and the Swedish Foundation for Strategic Research. Funding is also available from other funding bodies, such as the Wallenberg foundation, and for applied mathematical research, from the Swedish Agency for Innovation Systems (VINNOVA).

The Swedish Research Council funds project grants in Mathematics and Engineering Mathematics with a total annual budget of approximately 45 million SEK. In addition, approximately 5 million SEK are spent annually on funding individual positions and grants for younger researchers, such as individual postdoctoral grants and junior researcher positions. These calls are open to all fields. Adding to this, there is an annual call for senior research positions, intended for researchers less than ten years into their career, however, this call is for selected topics, and on average only one position every other year is in the field of mathematics.

Project grants constitute the heaviest funding post with an average grant size of about 650 thousand SEK per year. These grants allow for paying for research time for the project leader, hiring personnel (including PhD students and postdocs), travel costs and other expenses related to the research project. The selection of which projects or positions to fund is made by panels of experts who are themselves active researchers in the field.

The total budget for Natural and Engineering Science is about 1 billion SEK, of which approximately 4-5 percent is allocated to the panel for Mathematics and Engineering Mathematics for further distribution into selected research projects. The main part of the funding budget is allocated to researchers applying for grants (i.e. “bottom-up”), rather than allocated strategically.

The panel ranks, and suggests a budget for, the grants to be funded. The formal decision is then made by the Scientific Council for Natural and Engineering Sciences.

The Swedish Foundation for Strategic Research was established by the Swedish government in 1994 to fund university research in the natural sciences, technology and medicine, aiming at increasing Swedish competitiveness. Until about the year 2000, the main focus of the funding was on graduate schools and network programs. Some 1000 doctoral degrees were funded by the first 55 programmes. After the turn of the millennium, the Swedish Foundation for Strategic Research altered its priorities towards thematic calls.

The Swedish Foundation for Strategic Research attaches equal weight to scientific quality and relevance, where the former is ascertained by international experts in traditional peer review, and the latter is judged primarily with the help of experts from the industrial sector.

The current annual research budget is around 500 million SEK.

Instruments for support are

- strategic research centres
- research group grants (framework grants)
- individual grants, in particular:
 - future research leaders (every third year)
 - Ingvar Carlsson awards (for returning postdocs, about every third year)
 - mobility grants

Strategic research centres gather several research groups around an interdisciplinary topic. A typical grant may be 45 million SEK over a five-year period. Such a centre will also very likely involve some industrial collaboration. Research group grants are smaller, typically 3-7 million SEK per year over five years, and involves one or two groups. The future research leaders receive about 10 million SEK over five years, and the Ingvar Carlsson awardees about 3 million SEK over three years. In the 2002 call in Applied mathematics eight groups received five year grants for a total of 80 million SEK. In the 2004 call for strategic research centres two mathematics-oriented strategic research centres received five year grants totalling 22 million SEK each (for 2006-2010).

Over the past ten years the contribution from the Swedish Foundation for Strategic Research to research in mathematics (and including some adjacent areas) totals just over 200 million SEK.

Evaluation aims and methods

This evaluation concerns mathematical research awarded in the area of mathematics by either the Swedish Research Council or the Swedish Foundation for Strategic Research between the years 2002 and 2006. During this period, 123 research project leaders have been funded by one of the funding agencies. The major objectives are to evaluate the scientific quality in an international perspective and to identify strong and successful (as well as weak but important) research areas, and the strategic relevance of the research in question. Another aim is to identify the role of the Institut Mittag-Leffler with regard to its current form of organization and funding.

The selection of projects of the Swedish Research Council was based on projects awarded by the evaluation panel for mathematics (NT-R). The project leaders for these projects were asked to fill out a self assessment form and to select the primary, secondary and tertiary research area for their research (see Appendix 2 for the questions and areas). About 80 percent filled out the self assessment form (see Appendix 1 for the full list of projects).

A reference group of Swedish mathematicians gave advice on the selection of a Swedish researcher, not active at a mathematics department, to chair the international expert committee. Suggestions for experts that could serve on the committee were gathered from the reference group of the project, and also from the heads of mathematics departments and the researchers under evaluation.

As a complement to the individual self-assessments, the mathematics departments were invited to a hearing (see Appendix 3), in order to give the expert committee an idea of the environment where the project leaders were active. The meeting of the committee, as well as the hearings, was held at the Swedish Research Council during June 14 - June 18. Before the meeting in Stockholm, the expert committee was presented with the self-assessment data.

The expert committee was asked to provide an analysis of the quality and status of the research by evaluating the scientific value, quality and results of the research area, and by putting Swedish mathematical research into an international context. The committee was also asked to identify strong and successful as well as weak, yet important, research areas as well as potential research areas of great interest, and also to look at the balance between pure, applied and engineering mathematics. In addition, the committee was asked to identify the relevance for society in a short- to medium-term perspective of the research and to discuss the role of the research funding organizations, the Swedish Foundation for Strategic Research and the Swedish Research Council. Finally, the panel was asked to identify the role of the Institut Mittag-Leffler in Swedish mathematical research, taking into consideration the institute's current organization and funding.

The report of the expert committee is organized as follows: general remarks and recommendations are presented in Chapter 2, while assessments of the research areas are presented in Chapter 3.

In order to systematically grade the scientific quality of the research, the panel was asked to use the following scale:

Outstanding Outstanding research in an international perspective; of great international interest with broad impact and with publications in inter-

nationally leading journals; the grant holder is among the leading in the evaluated field of research from an international perspective.

Excellent Research at a very high international level; of international interest with impact within its field and with publications in internationally leading journals; the grant holder is competitive in the evaluated field of research from an international perspective.

Very good Research at a very good international level with publications in internationally well-known journals; the grant holder has a good international reputation within the field.

Good Research that is of good international standard and partially published in well-known international journals.

Insufficient Research of low international standard.

The grading of strategic relevance of the research was scored as:

Highest relevance

Very high relevance

High relevance

Medium relevance

Low relevance

The international expert committee

The committee was selected from a list of international experts, in turn suggested by a reference group of Swedish mathematicians, heads of mathematics departments where the project leaders were active, and the project leaders for the evaluated projects.

The chairman has not been actively involved in the actual research evaluation. Further information on the committee can be found in Appendix 4.

EVALUATION REPORT

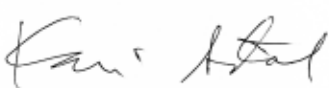
To
The Swedish Research Council
The Swedish Foundation for Strategic Research

At the request of the above-mentioned organizations, we have evaluated the Swedish research in Mathematics, funded by the Swedish Research Council or the Swedish Foundation for Strategic Research during the years 2002–2006. We take full responsibility for the judgments and the recommendations given in the report.

Stockholm, June 2010



Chairman. Prof. Ulf Danielsson
Uppsala University, Sweden



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University of Helsinki, Finland



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University of California at Santa Barbara, US



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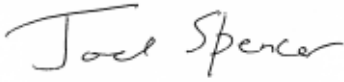
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The international expert committee.

2 THE EVALUATION COMMITTEE'S RECOMMENDATIONS AND COMMENTS

Swedish mathematics is certainly very strong in many areas, but the committee finds that the working conditions of mathematicians and of university researchers in general, are very difficult; the teaching loads are too heavy, and researchers must constantly apply for grants purely to ensure time for research. This is not good for research and the situation is particularly bad for younger mathematicians.

The committee also observed that the effective amount of funding for mathematics within the Swedish Research Council is very low compared with many other European countries and the USA in part because the demand for funding to reduce the high teaching loads absorbs funds that might otherwise be used for more creative purposes; while it is difficult to make direct comparisons of Swedish funding for mathematics with that in other countries because of the different structures for science funding, the committee still felt that the overall level of funding was low for the quality of the research programs deserving of support.

Sweden could make use of and afford much more mathematical research than is currently funded. Mathematics is instrumental for the development of new technologies, and investing in mathematics is indirectly investing in the future of Sweden.

2.1 Mathematics in Sweden: General observations

Sweden has a long and proud tradition of mathematical excellence. The projects reviewed by the Committee make it clear that mathematical research at the highest level continues in Sweden and that the global reputation of Swedish mathematics continues to be strong. The long-standing traditions of mathematical excellence in analysis and probability have been joined more recently by strong work in algebraic geometry, discrete mathematics and dynamical systems. Also, building on strength in traditional areas, and aided by research funding, applied mathematics has flourished in recent years.

The Committee sees two major challenges to the continued excellence of Swedish mathematics. First the committee finds that the working conditions of mathematicians and of university researchers are in general far

from ideal. Heavy teaching loads make maintaining strong research programs difficult or impossible. As a consequence, obtaining research grants becomes a necessity and much of the available research funding is used to provide research time which is a normal part of faculty appointments at major research universities in other countries. This situation has particularly negative effects on young researchers after their initial postdoctoral appointments, and on promoted professors who have no guarantee of the lighter teaching loads enjoyed by chaired professors. Overall, these conditions must significantly depress the levels of Swedish mathematics research. The uncertainty of research funding may also have negative impacts on teaching as the availability of faculty for instructional programs varies from year to year. The committee understands that the structure of faculty appointments has been actively discussed in the past, and urges that the Swedish Research Council take the lead in continuing these discussions with the goal of strengthening the research environment and making the most effective use of the research funds available.

The second major challenge is simply the overall level of funding available to support mathematical research in Sweden. Compared with other European countries and North America, the funding seems low. Additionally, much of the funding is used to free-up time for researchers with teaching appointments, and consequently funds available to support graduate students and postdoctoral researchers, and to provide general research infrastructure are limited.

The structure of academic appointments in Sweden and the limited availability of research funds present particular challenges for the recruitment and retention of world-class mathematicians and researchers in high-demand areas. The Committee noted a number of recent instances of very high quality researchers who could not be retained by Swedish mathematics departments. These losses and difficulties in recruitment should motivate all concerned with the future of Swedish mathematics to work together to resolve these issues to the benefit of Swedish mathematics and overall scientific development in Sweden.

2.2 Research funding

The primary sources of research funding for mathematics in Sweden are the Swedish Agency for Innovation Systems (VINNOVA), the Swedish Foundation for Strategic Research, and the Swedish Research Council. Though VINNOVA has funded a few mathematically-related projects in the past, its influence on mathematics funding is not great and the Swedish Foundation

for Strategic Research and the Swedish Research Council are the sources for most of the funded research in mathematics.

The Swedish Foundation for Strategic Research provides about 500 million SEK of research funding each year. A program of *Framework Grants* in 2002 targeted at the mathematical sciences supported projects that are part of the current evaluation. In addition, funding made available in 2005 for Strategic Research Centres supports the Centre for Industrial and Applied Mathematics (CIAM) at KTH Royal Institute of Technology and the Gothenburg Mathematical Modelling Centre (GMMC). These centres have had considerable success but face the familiar issues of survival after the initial funding has run out. These survival issues are particularly difficult in mathematics, where alternative sources of funding are not easily found.

The Swedish Foundation for Strategic Research also funds some mathematical researchers on an individual level, and some calls for proposals (for example, in information technology) provide opportunities in areas of the mathematical sciences. In general, the Swedish Foundation for Strategic Research directs its funding at young researchers and emphasizes interdisciplinary work.

The Swedish Research Council is clearly the largest public source for mathematics funding in Sweden. They disburse about 4000 million SEK per year with project research in mathematics receiving approximately 50 million SEK of this total (- 1.25 percent). Approximately 22 percent of the Swedish Research Council funding is used for large infrastructure projects and 23 percent is allocated strategically. The Institute Mittag-Leffler receives 2.5 million SEK each year from the infrastructure funds.

The Swedish Research Council Natural and Engineering Sciences panel has a single call for mathematics each year. Proposals are peer-reviewed and funds are awarded based on assessment of quality without regard to strategic goals.

The Committee offers the following observations regarding this primary funding process:

- As noted above, teaching loads tend to be very heavy, so funding success is not simply a nice “added extra” – it is crucial to the ability to carry out research
- The level of funding available to support mathematics is too low. The committee consisted of members from many countries and the feeling was unanimous that support for mathematics in Sweden compares quite negatively with other European countries and North America.
- The funding that is granted by the Swedish Research Council seems to be spent in roughly equal quantities on supporting PhD students and “buying

out” research time. The committee considers this to be an ineffective way to use research funding. Research time should be guaranteed to the researcher by universities. The grant money should be used to support PhD students, visitors, postdoctoral researchers, and other direct support for the research enterprise (such as organizing conferences and travelling).

- The Swedish Research Council states that it has a mission to support young researchers but, although many full professors can use their funding to carry out research, it is much harder for younger researchers to advance their careers. Some assistant professorship grants are awarded, but the number is low. It is strongly desirable that young researchers be better supported.
- There appears to be a lack of regular arrangements to fund the training of both graduate students and young researchers, and the committee felt that this was not in the best interests of mathematical research in Sweden.
- The total number of PhD students is not noticeably low, but the current system may lead to a tendency to fund postdoctoral researcher rather than PhD students, as they may produce “more research”. The Committee recognizes that postdoctoral researchers play a very important role in the development of a nation’s mathematical capability but is concerned that an appropriate balance be maintained to ensure that the necessary talent continues to be recruited for graduate training.
- At present it is not possible for more than one panel to provide support for the same grant. This makes success in joint applications and true interdisciplinarity very hard to attain. Many other funding agencies across the world, having recognized the advantages of multidisciplinary research, make special arrangements, for instance ad hoc panels, for dealing with multidisciplinary applications and initiatives. It may well be the case that the Swedish Research Council should give some consideration to this matter, as it is certainly likely that this will become an enduring theme in the future as large and complex cross-disciplinary projects become more commonplace and important.

2.3 Mobility

It has been noted above that the committee felt that recruitment and retention were quite serious issues for Swedish mathematics and for Swedish science in general. The committee further noted that many academics tend to spend their whole careers at the same institution, and there seems to be a relatively little mobility outside Sweden and the Nordic countries.

Though there are understandable reasons for this, the net effect is that there is a danger that the system could become inward-looking and be accused of being somewhat parochial. Of course, with the difficulties that have been pointed out, it may be that currently the only way to recruit and retain good researchers within the Swedish system is to “catch them when they are young”, as retention is more likely once the family unit has been firmly established in Sweden. It may be that, to be sure of both recruiting and retaining the best academics, some systemic changes need to be made to Swedish pay and conditions.

2.4 Infrastructure and research institutes

2.4.1 The Institute Mittag-Leffler

The Institute Mittag-Leffler is a central, internationally visible part of the Swedish national heritage in mathematics. Perhaps the oldest research facility dedicated exclusively to mathematics, the Institute has had a clear, documentable impact on mathematical research in Sweden and the other Nordic countries, as well as on mathematical research world-wide. As one measure of the impact on Swedish mathematics, more than half of the Swedish Research Council funded researchers reported participation in Mittag-Leffler programs during the period 2002-2009, with many citing important collaborations that began there.

The Mittag-Leffler institute is unique in its emphasis on groups of mathematicians working together in the most important areas of research with minimal distractions. This opportunity is particularly important for young mathematicians just beginning to establish ways of working and developing insight into the latest mathematical advances. The ability of the Institute to attract top mathematicians to work with young researchers in this setting more than justifies the expenditures necessary to maintain its long-term vitality.

The Institute helps make Swedish mathematics highly visible in the international scientific community. It attracts outstanding foreign mathematicians to Sweden who might not visit individual universities without the attraction of a Mittag-Leffler program and provides an opportunity for Swedish mathematicians to work in depth with the best mathematicians in the world. It provides an opportunity to introduce important areas of mathematical research that may not be well represented in the Nordic countries into the Nordic mathematical community.

However, the Institute faces serious financial challenges. While the Institute has an endowment held by the Royal Academy of Science, during the last several years the endowment itself has been used to balance the operating budget. Additionally, a portion of the Institute grounds was sold to raise funds to maintain the facility and there is still a critical need to upgrade participant housing if the Institute is to continue to maintain its place as a leading centre of mathematical activity.

The Evaluation Committee **most strongly recommends** the following:

1. The Swedish Research Council and other Swedish and Nordic scientific funding agencies should act quickly to stabilize the current operating budget and prevent further erosion of the Institute endowment.
2. The Institute should expand its programming into the summer months to increase its impact, make more efficient use of the facilities, and better justify financial commitments by the funding agencies and other sources of funds.
3. A Board of Trustees should be created including prominent members from outside mathematics to assist in communicating the value of the Institute to governments, funding agencies and foundations.
4. A major fund-raising effort should be developed to raise funds to upgrade the facilities.
5. The Institute should consider creating a board of business and industrial sponsors who provide financial support and assist the Institute in planning mathematical science-based short courses for researchers in business and industry.

The danger of losing this national treasure is real. The Evaluation Committee strongly urges that addressing the short-term financial shortfall and the long-term financial stability of the Institute be given the highest priority by the Swedish Research Council, other funding agencies, and the mathematical community of Sweden.

2.4.2 Other infrastructure needs

As far as other infrastructure needs were concerned, the evaluation committee felt that they had seen a rather mixed picture. It was not entirely clear that there was an overall strategy for the establishment of centres that related to mathematics. Often the establishment of a research centre involves the provision of “seed” funding from a research funding organization, which tapers off after a few years, and eventually runs out. By this time, the centre is expected to have grown sufficiently to be able to fund itself and

thereby ensure its own long-term future. The difficulty for mathematics (and in particular for pure mathematics) is that direct sources of income that might be used to sustain a research centre in the long term are not easy to come by. The business plan for centres to become viable in the long term is thus hard to formulate. Two examples (namely CIAM and GMMC) were discussed by the committee, but it was not clear in either case how the life of the centre could be prolonged after the initial funding runs out without a regular source of funding from the Swedish Research Council or the Swedish Foundation for Strategic Research. This was felt to be a problem for centres in general.

The committee also wish to comment that they had been interested in hearing details of high-performance computing (HPC) and supercomputing facilities that had been established at various individual universities. They felt that, at some time in the near future, the Swedish Research Council would have to develop a clear strategy on what would and would not be funded by the infrastructure component of the Swedish Research Council. It was noted that in many other academic communities worldwide the model whereby individual universities had their own large machine or other supercomputing facilities was not thought to be sustainable, and institutions were forming consortia to provide off-site facilities that exploited economies of scale and also naturally encouraged cross-institutional and multi-disciplinary research initiatives. It is possible that the future of HPC and Supercomputing in Sweden and other Nordic countries lies in a strategy that is based on premises such as this.

2.5 Cooperation and outreach

The Swedish mathematical community seems to be very open to collaborations, both within Sweden and with international partners. Most of the mathematicians who were funded between 2002 and 2006 show a very good record of research cooperation, mainly with foreign colleagues and many of their results and publications were generated in the framework of those collaborative projects.

The Mittag-Leffler Institute is a very good source of collaboration for Swedish mathematicians. The quiet periods spent there while participating in one or several programs facilitate the acquaintance of foreign or Swedish colleagues working in similar or related topics. Many publications and results have their origin in the Mittag Leffler Institute, and in some cases long-time collaborations were born or maintained thanks to the time spent there in close relations with other colleagues.

The committee has also seen with pleasure some interesting collaborative projects, such as between the departments of mathematics of KTH and Stockholm University in algebraic geometry, between KTH and Uppsala University in geometry, and in computational mathematics. The committee has also been informed of a very innovative cooperation within the department of Gothenburg University-Chalmers University of Technology, linking research in complex analysis and algebraic geometry.

Cooperation between academia and industry seems to be very good, especially in Gothenburg-Chalmers, Luleå-Narvik, Uppsala, KTH and Lund and the collaboration of Stockholm University and Karolinska Institutet in biostatistics is appealing.

There is a clear effort to move toward real life applications in mathematics departments all across the country. Interesting examples include the creation of the Fraunhofer Institute, and of the Modelling Centre in Gothenburg-Chalmers; and the strongly interconnected activities of several entities and groups in Uppsala (most of them linked to or administered by its Group of Computational Mathematics). Also, the presence of mathematicians in the decision-making process about intensive computing in Sweden is a very good sign of collaboration, with important output in many areas.

The particular arrangements made for a few prominent professors with positions abroad to have part-time positions and some investment in Sweden have created very good opportunities for young Swedish researchers at the PhD and post-doctoral level. Those young mathematicians have had access to first-class networks in fields which were not well represented in Sweden.

Regarding the outreach activities observed among Swedish mathematicians or within Mathematics departments, the committee has been very favourably impressed by the range of activities organized by most of the departments, or by their individual members. There are many courses aimed at high school teachers or students, and visits to high schools seem to be common. Exchanging views with school teachers or teaching them about recent developments in mathematics improves the level of mathematical knowledge among students. School teachers will then be able to transmit a different and more accurate view of mathematics, mathematical modelling and mathematical work, and can also convey modern views about mathematics, such as its usefulness and evolution. Additionally, approaching high school students can be a good source of recruitment for science. Students will then learn about mathematics and its development and applications from professional mathematicians, which will profoundly alter their perspective. They will be able to perceive science in general, and mathematics in particular, in a more realistic way which could encourage them to choose mathematics as their future career. In any case, it will give them a more accurate and interesting view about mathematics and mathematicians, and of the importance of

mathematics in the world in which we live. Conversely, working with eager young minds can be highly stimulating for researchers at any level.

An area in which the committee has seen a weakness is in the efforts made by the mathematical community to spread the message about the importance of mathematics in our modern world. Some mathematicians are involved in activities aimed at the general public, although nothing seems to be organized by mathematics departments in this direction. This weakness is not exclusively Swedish but can also be seen in many other countries. However, one might still expect better involvement in activities aimed at the general public, politicians and the press in the future. Other scientific communities have understood well the necessity to communicate their work and findings to the general public. Mathematicians have not, and unfortunately they sometimes consider this kind of activity as of lesser importance. Fortunately, several Swedish mathematicians are working in this direction and the committee encourages them to continue and the others to join them.

2.6 Impact

The impact of Swedish mathematics on the field of mathematics, and indeed on all the sciences including engineering, has been strong during the review period. The following is by no means an exhaustive list.

In numerical analysis and computational science the long tradition of research in analysis, partial differential equations (PDEs) and numerical analysis in Sweden has been brought to bear on a number of previously unsolved problems. In particular the theory of time discretization, finite differences and finite elements methods have been advanced significantly, and the applications to two-phase flow and complex fluids hold great promise for both technology and numerical analysis.

In dynamical systems theory, Swedish mathematicians have led the effort in proving the existence of strange attractors and the exploration of their statistical properties. Dynamical systems methods have been used to model complex environments, the motility of bacteria, and animal populations, with the inclusion of stochastic effects.

In algebraic combinatorics KTH has become established as a world centre. Probabilistic combinatorics is particularly strong in Uppsala, with strong interactions with probability and cryptography.

Gothenburg has been a centre for stochastic models in biology since long before it became a hot topic and it has been one of the few places that has mounted a consistent effort to penetrate the mathematically-resistant world of biology with serious mathematics having real relevance.

Creative efforts to train biostatisticians in Stockholm involving Stockholm University and Karolinska Institutet, and at Chalmers have successfully increased the pool of biostatisticians in the country. This work is central to biomedical research, and the program has had a significant positive effect on the institutions involved.

Mathematical analysis at KTH and Chalmers has been fundamental in the recent breakthrough of Stochastic Loewner equations, giving new understanding of two-dimensional random structures arising in areas such as random matrices, lattice models, Quantum Gravity, Diffusion Limited Aggregation (DLA) and Hele-Shaw flow. This has led to important developments in statistical physics and in understanding different critical phenomena.

The Swedish mathematician Jan-Olov Strömberg was one of the inventors of wavelets, the theory of which has been developed to a fundamental tool in numerical and applied analysis. It is being applied to a wide range of problems, which include radar imaging, fluid flow and combustion etc, and has been incorporated in the JPEG standard.

Three Swedish mathematics centres have created spin-off companies. At Karolinska Institutet and Stockholm University the Biostatistics group has created a spin-off company, as has the computational group at Uppsala University. Lund University seems to be unique among all the worlds' mathematics departments with more than 5 spin-off companies, some of which have already gone public.

The different groups in algebra and geometry in Sweden have had a significant impact in the different mathematical disciplines they represent – central and important disciplines which all are highly competitive and include a number of world-class mathematicians. To mention some examples, there is Ekedahl's and Faber's work on different moduli spaces, the deep results from the combined algebra/complex analysis group in Gothenburg, and the results obtained by Ekholm in symplectic and differential geometry.

In summary the impact of Swedish mathematics has been excellent and this bodes well for the future of mathematics in Sweden.

2.7 Specific needs and future directions of Swedish mathematics

Mathematics is a field where one never knows from what direction the developments having the greatest impact are going to come. Mathematics that is considered very pure today may become the best tool for applications tomorrow. Therefore it is very important to keep as many channels for mat-

hematics open as is possible and allow mathematicians to explore as many fields, pure and applied, as possible. At the same time it is important to retain the strengths that already exist and build up areas important for the future.

Sweden has considerable strengths in numerical analysis and computational science. These areas will to continue to increase in importance and have a wide impact on the future of science and technology.

Biology is the science with the most rapid current developments. There is a great need for mathematics to quantify biological processes. These include fields such as biostatistics, bioinformatics, dynamical systems and population dynamics, including cellular dynamics, and systems biology. The committee **recommends** a build-up of competence in these areas.

The apparent weakness of statistics (as distinct from probability and stochastic processes) in most Swedish departments of mathematical sciences points to a need to proactively support the development in this area. Successful training programs in biostatistics in Stockholm and Gothenburg may suggest a possible approach, and the strength of applied probability at many Swedish universities may help provide a base for this development. A further assessment by international statistical experts of the current situation might help focus development efforts in directions that will be most productive.

Biological applications will continue to drive the development of mathematics in many areas. Sweden has a strong base of mathematicians with biological interests and is in a good position to provide continued leadership in this area.

The committee encourages the continued strengthening of Probabilistic Combinatorics, particularly the study of large random structures and processes with connections to physical systems and social networks, including to the internet. The recent availability of data on web social networks is expected to lead to interesting mathematical developments in which the committee encourages the building of competence.

In the last ten years there has been a strong interplay between probability theory and complex analysis, which was motivated by interaction with physics. A major role was played by Stochastic Loewner Evolution, discovered by Oded Schramm, describing the only possible conformally invariant scaling limits of interfaces (domain walls) in critical 2D discrete models. As a result there is now an entirely new level of understanding of scaling limits of different discrete models in statistical physics. This will doubtless be one of the very topical trends in mathematical analysis, and the important role of Swedish mathematicians is expected to continue here.

The fields of algebra and geometry are sparsely represented in Sweden. Recent years have seen a rich interplay between geometry, algebraic geometry and theoretical physics. This is an area that is likely to continue to develop and the committee recommends strengthening the collaboration of the Mittag-Leffler and Nordita Institutes in this field with the development of Swedish talent in mind. The committee understands that the virtual Stockholm Mathematical Centre is also planning to have geometry as an important theme; the committee encourages this.

3 SUMMARY OF ASSESSMENTS OF SUBAREAS

3.1 Algebra and Geometry

Algebra and Geometry are not areas of traditional strength in Sweden, but during the latest decades it has grown and developed, and it now constitutes an excellent contribution to mathematics.

The overall coverage of these areas is however still rather thin. Important areas of contemporary research are simply not represented in the research supported by the Swedish Research Council, and others are represented by only a few persons, as detailed below.

Algebraic number theory is one of the interests of one person. Diophantine geometry, the topic of some of the great advances of the late 20th century, including the resolution of the Mordell conjecture by Faltings and the Fermat's Last Theorem by Wiles, is represented by one person.

Low-dimensional topology, currently the topic of greatest focus in topological research, is not represented. The theory of automorphic forms, particularly the Langlands program, is not represented. Riemannian geometry outside the theory of relativity is not studied. Two people can be considered as differential geometers in a broader sense. Three people work in areas related to representation theory. Logic is represented by one person.

Algebraic geometry is the best represented subfield. Most of the research in algebraic geometry is concerned with moduli problems - the description of the family of varieties satisfying prescribed conditions, for example the moduli space of algebraic curves of a given genus.

Algebraic geometry is a central part of mathematics, with roots going back to the beginning of the nineteenth century. There are connections to many neighbouring fields, including number theory and complex analysis, and nowadays also theoretical physics, the latest addition being the spectacular coupling of algebraic geometry to string theory.

It is especially good that many of these relations are manifest in the Swedish mathematical landscape where there seems to be an extensive cooperation between algebraic geometry and the neighbouring fields.

Stockholm University has had a strong tradition in pure algebra, but in recent decades the weight has shifted towards geometry. In their presentation for the committee, they stated that geometry (in a generalized sense) is

the dominant theme at the department, and that they will focus on it. The “generalized geometry” includes parts of complex analysis and theoretical physics, and the committee finds it very interesting. The committee **highly recommends** the Stockholm University to further explore the connections between geometry and physics by establishing a chair in mathematical physics. The Stockholm geometry group is performing **excellently**, working on themes central to contemporary mathematics, like operads, moduli of abelian varieties, real grassmannians, infinite Morse theory, motivic integration and D-modules in positive characteristic, amoebas.

At KTH there is a group of algebraic geometers doing **excellent** work. This is a rather new group in a KTH context, and there are plans to extend the group with a new position. There is also **excellent** work being done on the border between analysis and number theory. Moduli spaces of curves and their topological invariants is a hot topic in contemporary mathematics with connections to many neighbouring fields. Other parameter spaces are also studied, e.g., the Hilbert schemes and the Quot-schemes. This is also **very good** work, but it might benefit from a widening of scope. Finally, a third topic is toric geometry, where **very good** work is being done, with applications in combinatorics. The group as a whole is performing **excellently**, and encouraging it to grow is a wise policy for KTH. Researchers in related areas at other institutions (e.g., Linköping) are more isolated and would benefit from opportunities for increased interaction and cooperation.

The group doing algebra and geometry in Gothenburg ranges from people studying quantum groups to people working in several complex variables. There is a very fruitful and productive cooperation between people traditionally in complex analysis and people in algebraic geometry. Both groups are **excellent** by themselves, but the cooperation has given **outstanding** results. This is not in itself a new thing – algebraic geometry and complex analysis have always been close – but in Gothenburg the cooperation is unusually good. There is also **excellent** work being done in arithmetic and number theory, and one of the more modern inventions in number theory, Arakelov theory, is being explored in cooperation with the complex analysis group. The group is doing **outstanding** work.

There also seems to be some more isolated islands of people doing geometry in Sweden (e.g., in Linköping). **Good** work is being done, but the scope is somewhat narrow and the themes studied somewhat special. They might benefit from widening their interests and by cooperating more closely with the rest of the geometry community.

Representation theory (finite and topological groups, Lie algebras, Lie supergroups, quantum groups and related structures) has become a major area in mathematics, with links to physics, complex and harmonic analysis,

partial differential equations, number theory, algebraic geometry and other areas. Only four of the Swedish Research Council -supported investigators had representation theory as their focal activity, all in different directions. The level of the activity is **excellent**, but each worker is dependent on international contacts, and there are limited opportunities for interaction and collaboration.

3.2 Analysis

Research in mathematical analysis is done in every mathematics department in Sweden, but not every research group has received support from the Swedish Research Council or the Swedish Foundation for Strategic Research during the years in question. Here only the research that has received funding will be evaluated, which in the end may give a slightly distorted overall view.

Mathematical analysis has a very strong tradition in Sweden with a long and distinguished history of worldwide repute. In many areas of analysis Sweden still has mathematics departments which are world leaders in one area of analysis or another, but naturally the spectrum of mathematical research has become much wider. In analysis, outstanding research groups exist in Chalmers/Gothenburg, KTH (Stockholm) and Uppsala, with other departments e.g. in Lund or Linköping having small groups and Umeå or Stockholm University having strong individuals in the area. These successfully aim to work at high international level, but good quality research in analysis is done in practically every mathematics department in Sweden.

In brief, analysis is one of the main strengths in Swedish mathematics. In recent years there have been changes in major research directions within the field of analysis. To give one example of this, some 20-30 years ago complex analysis, harmonic analysis and Fourier analysis were much more central themes in Swedish mathematics. These still exist, but to a smaller extent than before. A great part of this research is redirected elsewhere in analysis.

The field still has many active leaders with excellent international cooperation. One worry, though, is that the country has lost quite a few of its younger stars, especially in analysis. This mobility provides a natural basis for international cooperation and a basis for young researchers to go abroad, but still it would seem advantageous for the country to create new means to attract back a few more of the emigrated highest stars. Perhaps some special professorships with stable source of funding to post docs and undergraduate students could be created to improve the situation.

Of the more traditional areas of analysis, analysis of several complex variables in particular remains **outstanding**. Here Chalmers/Gothenburg is a clear leader with a number of recent breakthroughs. There has recently been a strong initiative to apply analysis, via L^2 estimates towards analytical aspects of Complex Geometry, to algebraic geometry. Here highlights include results such as the analysis of complex Brunn-Minkowski inequalities, having strong connections to algebraic and Kähler geometry. The analysis of several complex variables has a strong tradition in many of the other Swedish universities as well.

Complex analysis of one variable and potential theory are fields of analysis which similarly have a strong tradition. Here, however, the present research volume is much smaller, and yet the area has in Sweden a good group of outstanding international top researchers, in particular at KTH. In this field there are high quality researchers also at Stockholm University and Linköping University funded by the Swedish Research Council. The research in complex analysis has received a strong impetus from interactions with mathematical physics. Here conformal mappings are key methods in topics such as Schramm-Loewner evolution, Hele-Shaw flows and random matrix theory, and in each of these promising areas, Swedish research has been very active.

Part of the traditional complex analysis and harmonic analysis moved over to dynamical systems, and KTH particularly grew one of the important centres worldwide. Unfortunately the school has recently lost some of its younger stars, but still remains an important player in the area doing from **excellent** to **outstanding** research. The highest relevance in present day in the dynamical systems research at KTH and Uppsala is currently focused on Hénon maps and Lorentz attractors, in particular around the existence of strange attractors and statistical and ergodic properties for Hénon attractors. Computer aided proofs have had also a strong role. An active school also exists in Lund, working on fundamental properties of dynamical systems as well as on interesting applications to mathematical biology and stochastic differential equations.

In classical harmonic and Fourier analysis, Sweden has been one of the strongest countries, but today these areas have lost some of their prominent status in the country after the moves towards the important contributions to dynamical systems. Although Swedish researchers did some of the earliest work on what has become the theory of wavelets, the mature version of the theory was developed elsewhere. However, wavelet ideas have returned to Sweden, and are now incorporated in software produced by groups in numerical analysis and scientific computation.

Nowadays, the field of harmonic analysis is represented in Sweden, e.g. in the form of non-commutative harmonic analysis, analysis of Lie groups and homogeneous spaces, while Fourier analysis concentrates more on pseudo-differential operators and various applications to PDEs. Here the research is conducted at an **excellent** to **outstanding** level at KTH and Lund, with **very good** to **excellent** researchers also at Chalmers and Linköping.

Among the more recent openings in mathematical analysis one finds global analysis, including topics from non-commutative ergodic theory and spectral theory for differential operators on manifolds, to metric geometry. Here in particular there is an **excellent** level of work being done at KTH, with **very good** groups of young researchers also at Chalmers University.

Sweden also has a strong history also in other classical areas such as functional analysis and operator theory. Today in these areas there are only a few researchers, at least within the grants funded by the Swedish Research Council or the Swedish Foundation for Strategic Research. **Very good** and **excellent** important research is being done, in particular nonlinear functional analysis for PDE, although this has partly suffered from cuts in funding.

Analytic number theory has, equally well established roots in Swedish mathematics, but has now witnessed the appearance of **outstanding** young researchers in Uppsala and at KTH. Here one also finds connections to topics such as quantum unique ergodicity and other spectral questions related to number theory.

Within partial differential equations, Sweden has a large spectrum of research ranging from **very good** up to **outstanding**, including computational or industrial applications and to a variety of theoretical studies in PDEs. The topic of linear PDEs has been one of the strongest areas of Swedish mathematics, with Sweden at least for a while having leadership in the world, with Riesz, Hörmander and Gårding. The area of nonlinear PDEs is now much more alive, following in the footsteps of Swedish researchers such as Dahlberg or Aronsson. In this topic many departments are well represented. The topics here range from the study of general variational methods and applications developed in Uppsala and at Stockholm University to the study of fluid mechanics and water waves, the kinetic theory of systems with large number of particles, Hamilton-Jacobi equations, the equations of general relativity and the study of free boundary problems developed at KTH, Chalmers, Uppsala, Linköping and Karlstad. Some attempts to build research strength in nonlinear PDEs have suffered from departure of young faculty (Lund) but others (Umeå, Chalmers) hold great promise.

The above brief description of research in mathematical analysis in Sweden shows clearly the **excellent** to **outstanding** level of the area and its **highest strategical relevance**. The committee sees it as important for the

development and future of science in Sweden that the funding from the Swedish Research Council, the Swedish Foundation for Strategic Research and other Swedish sources is sufficient to support the whole spectrum of analysis, ranging from the classical high quality research to the newly emerging fields. The support of young investigators is particularly important, combined with an effort to attract a number of Sweden's emigrated top young researchers.

3.3 Applied and engineering mathematics

For many years Mathematics in Sweden was largely based around excellence in various sub-branches of Pure Mathematical analysis. More recently, however, various fields in Applied Mathematics (in its largest sense) have come to prominence. The committee judged that great progress had been made in the fields of Applied and Engineering Mathematics, both in theoretical and application-driven areas. It was also clear that, though the many natural interfaces with Numerical Analysis (see 4.8 below) had been exploited to advantage in a number of Swedish institutions, it also seemed that in a few institutions Numerical Analysis was still thought of as being more properly part of Computer Science. This undoubtedly led to some missed opportunities as synergies were consequently harder to exploit. In some institutions a very entrepreneurial approach had led to the creation of spin-offs and real contact and interaction with a range of important industries. The arrangements for distributing the financial reward that might arise from spin-offs seemed very odd to the committee, as the universities seemed to make no money at all from any successes, but of course this is an internal matter for the Swedish University system. Finally, in giving an overall rating of **very good** to **excellent**, the committee wish to make clear that, to arrive at this rating, a rather rough averaging process has necessarily taken place, since the fields of Applied and Engineering Mathematics cover a huge range of different sub-disciplines (perhaps more than any of the other areas that were being evaluated).

The Gothenburg/Chalmers union of mathematics departments has, in the committee's view, produced a great deal of **excellent** (and some **outstanding**) work. Probability and Statistics is very strong and has made an important contribution to the overall effort in applied mathematics. There is also good engagement by groups working in mathematical biology and numerical analysis. Two centres - Fraunhofer and the Gothenburg Mathematical Modelling Centre (GMMC) - are also making a great contribution to applied mathematics. The work of the Fraunhofer institute shows enga-

gement with a wide range of industries and confronts real industrial problems using mathematics that also has theoretical importance. GMMC has made alliances with systems biology, and worked on other problems including ship safety, gene expression and portfolio optimization. There is also a successful Centre for Applied Mathematics and Statistics, and evidence of excellent publications and other outputs in the areas of both Biomathematics and Optimization. Overall the research is **excellent**.

Karlstad works to some extent in partnership with Uppsala and Mälardalen. The research groups in Applied Mathematics are relatively small and a somewhat isolated, but there is **excellent** work being done in granular flows and kinetic theory. Probably the most important issue for Karlstad will be to carefully select areas of possible future excellence, and invest in them selectively. In summary, much of the work that takes place in Applied and Engineering Mathematics is **very good**, and an encouraging proportion is **excellent**.

The committee saw much evidence of Applied Mathematics of a very high standard at KTH. A small but **outstanding** group carries out impressive work on multiphase flow and suspensions. The CIAM centre that has been established has also clearly carried out some **excellent** work, but its future is unclear and a careful strategy needs to be formulated to ensure that it continues to survive after its initial funding has expired. The Optimization and Systems Theory section of the Department has carried out a great deal of **excellent** work and has a number of strong members. Its work is also being applied to a number of important practical problems as well as being strong in a purely theoretical sense. In summary, the committee felt that applied mathematics at KTH, though largely of a traditional sort, attains a general standard of **excellent**, and also includes parts that are **outstanding**.

It is clear that high quality work is being carried out in a number of areas of Applied and Engineering Mathematics at Linköping. Some areas, such as particular kinds of applied optimization, have clearly made an impact, but they are somewhat specialized and have not attracted large quantities of research funding. **Excellent** work is also being carried out in other specialist areas such as p -Laplacian theory, and there is also significant activity in potential theory, inverse problems and aerodynamics. Overall, Applied Mathematics at Linköping may be regarded as being **very good**, and some more specialist parts are clearly **excellent**.

The main group in Luleå specializes in mathematical homogenisation. This work has a good theoretical background, but the committee had no doubt that the main strength of the group lies in using its work to make important contributions to real industrial and engineering problems and products. This is notable, as homogenisation is a subject whose specific goal

is applications, and this is all too often forgotten. The Scientific Computing group also interacts with the Applied Mathematicians and strikes a pleasing balance between theory and applications. The work being carried out on the boundaries between pure and applied mathematics is also notable. Overall, the research is **very good to excellent**.

The field of probability is strongly represented in Lund, and has clearly had an impact on the effort in Applied Mathematics. One gets a very contemporary feel about the Applied Mathematics that takes place at Lund – there are good connections with technology and good overall technology transfer, and this may be the future of the subject. Statistics for stochastic processes is strong, and the department seems to truly be a part of the international research community. The Computational Modelling Group has carried out **excellent** work, and there is also a strong mathematical imaging group. Work in mathematical finance has had an impact, and work on stochastic processes in medicine has produced some good outputs. Finally, the numerical analysis group is well engaged with many practical problems. Overall, the committee found a great deal of **excellent** work being carried out at Lund.

The department in Mälardalen here is very small and appears to be somewhat isolated from other Applied Mathematics departments in Sweden. It is also clear that members in the department work in a range of rather non-standard areas. The Centre for the Study of Cultural Evolution (which is based in Stockholm) is partly run at Mälardalen and has received funding from the Swedish Research Council. Though this work is both interesting and unusual in interfacing with so many other culturally-related disciplines, it is as yet unclear what mathematical impact this work has made and how much mathematical impact this department is likely to have. The department has issues in attaining critical mass, and so at present can only be regarded as **good**.

The committee found evidence of interesting work in applied mathematics at Umeå University. The numerical analysis group was traditional and focussed on linear problems, but had produced some very useful results that could be applied to real problems. The applied mathematicians clearly collaborated with the numerical analysts, and there was also evidence of valuable finite element collaboration with physicists. The cooperation and exchange of Applied Mathematics clearly took place on an international level and there was also interesting work in some environmental and biological applications. Overall rating, the applied mathematics in Umeå is **very good to excellent**.

The computational group at Uppsala is clearly very strong and produces a great deal of **excellent** work that is not only good from a theoretical point of view, but also has impact on real problems. CIM, the Centre for

Interdisciplinary Mathematics, engages both other mathematically-related disciplines and industry. There are many strong areas and many strong researchers, and the committee had no hesitation in classifying the general work in Applied and Engineering Mathematics as **excellent**.

3.4 Combinatorics and Discrete mathematics

Discrete Mathematics (which will be used interchangeably with Combinatorics) ranks as one of the great prides of Swedish mathematics. This is a rapidly growing area in which the theoretical and applied aspects are completely interwoven. If one goes back only forty years this area was regarded somewhat disparagingly as puzzle mathematics or worse. Since then the methodologies have evolved and been solidified. The rise of the computer, for which Discrete Mathematics is a natural setting, has had a powerful influence. Also critical have been particular individuals such the Hungarian Paul Erdős and the American Gian-Carlo Rota.

Sweden has been fortunate to have a world leader in Algebraic and Topological Combinatorics. KTH is therefore a powerhouse in Algebraic Combinatorics and can plausibly be argued to be the number one place in the world. There is a constant stream of visitors to KTH, the members of the group are highly active and their work naturally connects to Algebra and Geometry.

Probabilistic Mathematics (the asymptotic study of large discrete random objects) is a second great strength of Swedish mathematics. This is most evident in the study of Random Graphs (in which we include other random structures). The leading figure is in Uppsala and has been enormously influential in Probabilistic Combinatorics. Random Graphs are connected with percolation phenomena and there is extensive work being done analysing behaviour at or very close to critical points. This also connects to Branching Processes in Population Dynamics and elsewhere, as studied particularly at Chalmers. The work on random structures involves deep statistical considerations. For many natural random variables, such as subgraph counts, it is relatively easy to calculate mean and variance but showing Gaussian behaviour is challenging. The interactions between Probability and Probabilistic Combinatorics are profound and it can safely be said that Sweden's greatness in Probability has led to its successes in Probabilistic Combinatorics.

The solution of the Parisi Conjecture by two Swedish mathematicians (now at Chalmers and KTH) was a great triumph. They were able to analyse the job assignment problem, a basic optimization question. Parisi, an Italian physicist, had used deep insight from Mathematical Physics to conjecture

expected behaviour. The connection between Mathematical Physics and problems in random structures (Random k-SAT being another prime example) is at a critical moment. Today physicists have techniques such as the replica method which mathematicians have not yet been able to place on a firm footing. From a historical perspective one can hope for great breakthroughs when this is done. There are groups and individuals at Uppsala, Chalmers and KTH that are poised to make significant contributions.

Logic is not well represented in Sweden. Stockholm University will have a new position, replacing a distinguished retiring professor. There are many exciting connections between Logic and Computer Science and the committee would recommend exploring that direction. There are only a handful of logicians in Sweden, but a country of this size cannot be strong in all directions and we make no recommendations.

Overall, Discrete Mathematics in Sweden is **Outstanding**. KTH is **Outstanding**. Uppsala is **Excellent** to **Outstanding**. There are also **good** smaller groups at Chalmers, Umeå and Linköping University.

3.5 Mathematical physics

The definition of Mathematical Physics could be a subject of discussion, and when looking at the mathematicians funded by the Swedish Research Council and the Swedish Foundation for Strategic Research during the period 2002-2006, one sees that the definition is not the same for everybody. One could say that this field is the one concerned with the mathematics which studies models and problems coming from Physics. If we take this definition, then large parts of analysis, nonlinear partial differential equations, geometry and probability and even statistics would be concerned. Actually, in Sweden many so-called Mathematical Physics departments or groups seem to be included in Physics more than in Mathematics. In some sense, what in Sweden many call Mathematical Physics seems to correspond to what in other countries is called Theoretical Physics. To be precise, the text below is concerned with the community of mathematicians working in Mathematics departments but dealing with models coming from Physics.

Historically Mathematical Physics has been a very important topic worldwide because, for many centuries Mathematics advanced when trying to solve physical or mechanical problems, i.e. when trying to model and understand reality.

Looking at the number of people who received the Swedish Research Council grants in the current evaluation exercise period gives the impression that the field of Mathematical Physics is not very well represented in Sweden.

There are very good researchers in different places and in different sub-fields, but not many. The main places where one can find people working in Mathematical Physics, as a primary or secondary field, are KTH, Stockholm University, Chalmers, Lund and to some extent in Karlstad. In this evaluation, only 3 researchers declared Mathematics Physics as their primary field of work, two being specialists in nonlinear partial differential equations in general relativity, and one working on supersymmetric matrix models, singularity formation in relativistic extended objects, and on the study of symmetry for boson systems. However a total of 18 persons declared Mathematical Physics as a relevant field for them. Most of them are analysts (experts in dynamical systems, partial differential equations, special functions, etc) or geometers who have an interest in problems and models coming from Physics.

It then seems that mathematical physics as such is not a very important topic in Sweden, because no department has a well-defined group in this field. Nevertheless, there are **excellent** researchers here and there, but they are scattered across the country and they function mostly in networks which are almost fully international. Also, many people who declare themselves primarily in the fields of Analysis or Geometry or Probability can be linked to this field at different levels of involvement.

Concerning the interaction of Geometry and Physics, one finds mathematicians in several Swedish departments working on mathematical aspects of quantum field theory and string theory, and also on Hamiltonian systems. This is the case in Stockholm University and in Uppsala. Regarding Stockholm University, the committee highly recommends that the department further explores the connections between Geometry and Physics by establishing a chair in Mathematical Physics.

In KTH, one of the strongest departments in this area, there are people working in dynamical systems, Hamiltonian and chaotic dynamics, ergodic theory, general relativity, Hele-Shaw models, which have a strong relation with scaling laws of discrete models in statistical physics, random matrix theory, supersymmetric matrix models, singularity formation in relativistic extended objects or the study of symmetry for boson systems, fluid dynamics, two-phase flows and models for coarse-grained molecular systems, spectral theory, spectral geometry, etc. So, even if for many of these people the main subject is not Mathematical Physics, their work has a lot of connections with physical models and in that sense they could count as being at least partially in Mathematical Physics.

In the area of nonlinear PDEs one finds many **excellent** researchers in several mathematics departments in the country, who could count in one way or another as mathematical physicists. They work in general relativity (KTH and Chalmers), on Hele-Shaw flows (KTH), integrability issues (Luleå),

kinetic theory for systems of many particles, granular flow, etc. (Gothenburg-Chalmers and Karlstad), and in fluid dynamics and water waves (Linköping).

Among the Swedish probabilists, there are some people working on statistical mechanics, a field that can also be considered as part of Mathematical Physics, including at Chalmers University and KTH.

Also at Chalmers University, there is a very interesting project on the link between special functions (elliptic hypergeometric functions) and quantum field theory and another one concerning some applications of Algebra to the study of quantum groups.

Finally, other important themes are operator theory and spectral theory. These can be found in many departments, including KTH, Stockholm University, Lund, Linköping, Uppsala and Gothenburg-Chalmers. The research done in this area is overall at an **excellent** level, and in some cases it is **outstanding**.

In summary, in Sweden there are a small number of **excellent** and **outstanding** people with a primary interest in Mathematical Physics and a larger number of Swedish mathematicians working in fields which are linked in one way or another to Mathematical Physics. However, the sub-fields are numerous and there is no well-defined group anywhere working in this area. The overall level is **excellent** and in many cases comparable to the best that can be found in other countries. This field could become more influential with increased support.

3.6 Mathematical statistics and probability

Mathematical Statistics

In the organization of the Swedish Research Council and most Swedish Universities, Mathematical Statistics is interpreted to include probability and stochastic processes in addition to statistics, including biostatistics. In addition, important areas of application including telecommunications, population biology, and finance are well represented among faculty and the Swedish Research Council projects classified in this area.

Probability and Stochastic Processes

There seems to be a general consensus in the reports the committee received that research in probability and stochastic processes is much stronger and better established in Sweden than research in statistics. Indeed, research in Sweden in this area is **outstanding**. There are important, internationally

recognized centres of excellence in Gothenburg, Stockholm, Lund and Uppsala, with a **very good** representation in other Swedish Universities as well. Branching processes and other models of population dynamics, random graphs and related work in discrete probability, and particle systems and percolation theory are particularly strong. There is an **excellent** program in the modelling of epidemics at Stockholm University that appears well integrated with the statistics program there. KTH has a **very good** active group in stochastic analysis related to finance.

The Stochastic Centre in Gothenburg is the largest group working in probability and stochastic processes in Sweden and among the largest and strongest in the world. For the past 15 years, the work of this Centre has been supported by a grant from the Swedish Foundation for Strategic research and major funding from other sources. These large grants have led to **outstanding** development of the Centre and have greatly enhanced its impact on the Swedish and international research communities.

Overall, Swedish research in this area is of the **highest relevance** to many areas of application, and excellent connections have developed between researchers in core stochastic areas and the areas of application.

Statistics

Very little of the funding in mathematics by the Swedish Research Council is supporting research in core areas of statistics. Apart from two large biostatistics training grants discussed below, the Swedish Research Council is supporting **excellent** statistical work related to epidemics and genetics and spatial and environmental statistics, and **very good** general statistical work related to stochastic processes, but little else. The minimal level of funding allocated to statistics seems to reflect the relatively weak position of statistics within mathematical statistics groups at Swedish universities. This understanding was reinforced by comments regarding the difficulty in finding staff needed to teach statistics at some universities. While applied statisticians can be found in social science departments and business schools, statistical methodology is also of **very high relevance** to many areas of science, particularly in the light of the increasing complexity of experiments and the massive amounts of data collected. The need to develop core statistics would seem to be a priority.

Biostatistics

The development of biostatistics in Sweden has apparently lagged even farther behind the development of mathematical statistics and other areas

of applied statistics. Two of the reports the committee received presented results of grants specifically made to address this issue. A very successful biostatistics training program was developed at the Karolinska Institutet and Stockholm University over the past seven years. This program has involved at least ten faculty members from the two institutions and has produced eight PhDs with six different advisors in the last ten years. Graduates from the program have taken academic and industrial positions in Sweden and elsewhere. The special funding has now ended and it is not clear that the momentum developed in this excellent program will be maintained. A **very good** smaller program at Chalmers produced three PhDs, all of whom have taken positions at Chalmers.

While these programs represent significant progress, a number of Swedish universities are having difficulty filling positions in biostatistics at least in part because Swedish academic salaries do not compare well to salaries paid in industry. Biostatistics is of the **highest relevance** to research in the medical sciences and further development in this area should be a high priority.

3.7 Numerical analysis

Numerical analysis or computational science was introduced very early into Sweden. One of the very first computers was built in Sweden and numerical analysis started to grow accordingly. Strong centres appeared at KTH, Lund, Uppsala and Chalmers, and then also at Linköping and Umeå including an impressive list of outstanding, world-leading scientists, such as Fröberg, Dahlquist, H-O. Kreiss, Thomee and Björck to name a few. Time discretization and difference methods were the theme at KTH and Uppsala, Finite element methods (FEM) was strong at Chalmers and numerical linear algebra in Linköping and Umeå. Computer science departments in Sweden partly grew around this tradition, and even today part of numerical analysis is located outside mathematics departments.

KTH and Uppsala enjoy today, after a generation change, much cooperation and their best science continues to stay at an internationally leading level. The research groups active at these universities are **outstanding**. The research conducted by these groups is clearly of the **highest strategic relevance**.

Umeå has kept a rather steady profile and can be called **excellent** while Chalmers, Linköping and Lund are partly undergoing a transition period but are still **very good**. The **strategic relevance** of the research done at all of these universities is **very high**.

The research at Luleå can also be deemed to be of **medium to high strategic relevance**.

Overall rating of numerical analysis in Sweden is **excellent to outstanding**.

Numerical analysis and scientific computing are within NADA at KTH which is administratively in a different school from mathematics. Starting from the early 1960's, the department grew into one of the leading centres in the world in the time discretization of ordinary differential and differential algebraic equations. Particular strengths of the department were a good command of analysis (in particular in function theory), and a true interest in collecting important practical application problems (stiff equations) to guide the direction of the research.

The scope widened into hyperbolic problems and having a senior investigator staying partly in the US, involvement in the newest trends has been excellent. A young investigator instigated research into stochastic differential equations in a successful and original way, and NADA has been able to recruit two excellent younger researchers from the Courant Institute. The research is **excellent to outstanding** and, with the new younger researchers, it is likely that it will continue at this unusually high level.

There are strong links between NADA and the Division of Scientific Computing in Uppsala, which will be commented on below.

As at NADA, the roots at DSC (Division of Scientific Computing) at Uppsala University are in keeping with Sweden's strong tradition in analysis. This environment has now been transformed into a very impressive blend of numerical analysis and computational science. Some of the research deals with numerical simulations of two-phase flows and complex fluids. These are some of the most challenging and important problems in contemporary fluid dynamics. The research is a team effort involving several investigators and the collaboration between two of the investigators seems particularly fruitful. This is first rate research of great scientific importance. For example the flow through the combustion chamber of a jet engine is a two-phase flow and there is great interest internationally in the results both for theory and applications.

The current research also addresses the numerical analysis of compressible gas dynamics, wave propagation and two-phase flow. The effects of artificial (numerical) viscosity on compressible dynamics and of absorbing boundary layers in wave propagation and capillary effects on the phase boundary in two-phase flow are addressed. These are some of the most difficult problems in modern applied mathematics. The results are truly impressive and of great scientific importance. The collaboration of the two investigators at Uppsala and KTH is clearly **outstanding**. It is worth mentioning that DSC

had a joint Centre of Excellence together with groups at NADA 1995-2005, supported by both Uppsala University and KTH, VINNOVA and industry. Such cooperation has widened the profile successfully into many directions of scientific computing.

The joint programs of KTH and Uppsala promise to retain in Sweden one of the best centres of numerical analysis/computational science in the world and continue its strong tradition in this field. This is obviously **outstanding** research and of **very high relevance** to the particular application fields.

Mathematical activities seem to have been integrated in a successful way in Gothenburg. The FEM school at Chalmers (Thomee and Johnson) in numerical PDE's was one of the leading schools in the world in elliptic and parabolic problems.

Today the emphasis is in a new direction. There are now two new institutional efforts, Fraunhofer (2000) and GMMC (2005), where important interdisciplinary research is conducted. Numerics and computational mathematics have gained from one investigator moving from mechanical engineering, one investigator shifting towards stochastics and a younger researcher taking numerics for inverse problems. This is all very promising. The research is **very good** and **highly relevant**.

The long tradition in Umeå in numerical analysis has concentrated on numerical linear algebra with special attention to computer architecture.

The senior scientist at the Computing Science Department has been central in establishing the High Performance Computing Center North (HP-C2N). The research has been persistently on high level, with well functioning international cooperation. The software products are of high quality and of very high relevance to the scientific community, including a novel QR algorithm for eigenvalue calculation with parallel computers. The group has recently widened its scope by recruitment from Uppsala. There is also some numerics-related activity at high level in the mathematics department. This research is **excellent** and its applicability in neighbouring experimental sciences and industrial engineering makes it of the **highest strategic relevance**.

Numerical analysis in Linköping cannot be discussed without mentioning Åke Björck who for a very long time was an international leader in numerical linear algebra, in particular in least square problems. One investigator has continued this fine tradition at a high level and directed the research towards matrix methods in data mining and pattern recognition.

The group in scientific computing has recently recruited a new member and the scope has thus widened to numerical PDEs. In addition to numerical analysis in Scientific Computing, the Research School in Interdisciplinary Mathematics has projects with strong numerical components. The research is **very good** and of **high strategic relevance**.

Numerical analysis in Lund grew around Carl-Erik Fröberg who for example started the journal BIT, which is now celebrating its 50th birthday.

Today mathematics in Lund is collected in the Centre of Mathematical Sciences which contains three different units in itself. Of these, scientific computation is strong in LTH but there is also interesting research in computational mathematics at the science faculty.

Current research topics continue the tradition of time discretization of the KTH school at a high level with emphasis, for example, step size control in Hamiltonian systems and on DAEs. The group has very good contacts in the field. In addition to this there is a strong group in imaging.

Lund has a very lively environment for interaction with industry, which is reflected also in the number of spin-off companies which have grown out from mathematics. The research is **very good** and of **high strategic relevance**.

The research in Luleå addresses the homogenization of partial differential equations with rapidly varying coefficients. The research group that encompasses Luleå Technical University, Narvik University College and Uppsala University has become strong in homogenization. It has particularly close connections to applications and industry and the mathematicians work closely with engineers working on applications of the theory. This research group represents the ideal of how collaboration among mathematicians, between different universities and across departmental boundaries can be. It was formed under the leadership of Lars-Erik Persson and it represents the most beneficial research environment that one can wish for. It is a highly desirable situation to have and is somewhat unusual for Sweden and Scandinavia. The results are obvious when one looks at research productivity and the number of new PhDs produced by the group. This research is **very good** and clearly of **medium to high strategic relevance**.

APPENDIX 1: LIST OF PROJECTS

Chalmers University of Technology

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2003-5214	Albin, Patrik	VR	Asymptotic and Related Properties for Stochastic Processes, with Risk Applications	1 316 400
2004-991	Andersson, Mats	VR	Multivariable residue calculus and Operator theory	1 419 000
2004-3590	Andreasson, Håkan	VR	The global structure of the solutions to the Einstein-Vlasov system.	1 419 000
2006-3280	Berman, Robert	VR	Bergman kernel asymptotics in complex and CR geometry	3 615 000
2002-5521	Berndtsson, Bo	VR	dbar-equations, Bergman kernels and Morse inequalities	1 460 160
2006-2627	Berndtsson, Bo	VR	Subharmonicity and curvature	1 950 000
2002-5314	Häggström, Olle	VR	Percolation and phase transitions in statistical mechanics	4 788 160
2004-4272	Jagers, Peter	VR	The Gothenburg Stochastic Centre	7 099 000
SSF 8	Johnson, Claes [†]	SSF	Computational Mathematical Modelling: Chalmers' Finite Element Centre	6 000 000
2006-2647	Jonasson, Johan	VR	Discrete stochastic models	1 732 000
2002-5353	Larsson, Stig	VR	Numerical modelling of viscoelastic material using integro-differential equations	905 840
2006-3999	Larsson, Stig	VR	Numerical methods for stochastic partial differential equations	1 950 000
2002-5547	Månsson, Marianne	VR	Part 1. Models for non-overlapping particles/Part 2 Compound Poisson processes	838 240
SSF 18	Nerman, Olle	SSF	Bioinformatics Research Programme	7 425 000
SSF 12	Nävert, Uno	SSF	Fraunhofer-Chalmers Research Centre for Industrial Mathematics	10 000 000
2002-5780	Patriksson, Michael	VR	Mathematical models and optimization methods for robust optimal design	973 440

Chalmers University of Technology (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
SSF 2	Rootzén, Holger	SSF	Gothenburg Mathematical Modelling Center	26 000 000
2004-4263	Rosengren, Hjalmar	VR	Elliptic quantum groups and elliptic hypergeometric series	1 419 000
2003-5448	Salberger, Per	VR	Counting rational points on projective varieties over number fields	1 316 400
2006-2697	Salberger, Per	VR	Counting solutions to Diophantine equations.	1 800 000
2003-5355	Särkkä, Aila	VR	Statistical models for the influence of trees on the abundance of under-story vegetation species in boreal forests.	198 100
2006-3998	Särkkä, Aila	VR	Development of spatio-temporal models for tree growth	1 950 000
2002-5273	Turowska, Lyudmila	VR	Operator synthesis: measure-theoretical approach	1 257 360
2005-6470	Turowska, Lyudmila	VR	Operator synthesis and its applications	1 623 000
2003-5354	Wennberg, Bernt	VR	From systems of particles to kinetic equations	1 316 400
2006-3229	Wennberg, Bernt	VR	Kinetic limits of many particle systems	1 950 000
2006-3995	Wästlund, Johan	VR	Stochastic models of optimization problems	1 722 000
2003-5415	Zhang, Genkai	VR	Analysis on symmetric domains and representations of Lie groups	1 316 400
2006-3231	Zhang, Genkai	VR	Harmonic analysis and Radon transform on symmetric spaces. Representations of Lie groups	1 599 000

Gothenburg University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2006-3232	Rychlik, Igor	VR	Stochastic analysis and computation for ocean waves	1 950 000
2005-3072	Stolin, Alexander	VR	Double cosets in quantum groups and number theory	1 660 500

Karlstad University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2003-5357	Bobylev, Alexander	VR	Mathematical problems and methods in nonlinear kinetic theory	1 539 000
2006-3404	Bobylev, Alexander	VR	Mathematical problems and methods in nonlinear kinetic theory	1 950 000

Karolinska Institutet⁴

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-2964	Pawitan, Yudi	VR	Statistical genetic modelling of multivariate binary traits from population-based family data.	1 782 000

Royal Institute of Technology

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
SSF 4	Amberg, Gustav	SSF	Mathematical Theory and Simulation Tools for Phase Transformations in Materials	13 000 000
2002-5403	Benedicks, Michael	VR	Chaotic behaviour and positive Lyapunov exponents for some dynamical systems and matrix cocycles	1 460 160
2003-5371	Björner, Anders	VR	Algebraic Combinatorics	1 539 000
2006-3285	Björner, Anders	VR	Algebraic Combinatorics	1 267 000
2005-2583	Chacholski, Wojciech	VR	Homotopy meaningful constructions in algebra, combinatorics and topology	1 660 500
2002-4858	Dahl, Mattias	VR	The Dirac operator and scalar curvature on Riemannian and Lorentzian manifolds	3 158 064
2006-3539	Di Rocco, Sandra	VR	Applications of toric geometry to Combinatorics and Numerical Analysis.	1 944 000
SSF 3	Engquist, Björn	SSF	Advanced computing techniques for new discoveries in science and engineering	13 500 000

Royal Institute of Technology (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2004-3694	Faber, Carel	VR	Moduli spaces, Hilbert schemes, and their cohomology	3 516 000
2002-5588	Faber, Carel	VR	Enumerative geometry of moduli spaces of curves	1 460 160
2003-5440	Forsgren, Anders	VR	Large-scale nonlinear optimization	1 316 400
2006-3238	Forsgren, Anders	VR	Large-scale nonlinear optimization	1 865 000
2004-2010	Gustafsson, Björn	VR	Potential theory with applications	1 419 000
2004-4302	Gustavsson, Katarina	VR	Numerical Simulations of Fiber Suspensions	1 508 400
2003-5375	Hedenmalm, Håkan	VR	Bergman space methods in analysis	2 612 400
2006-3258	Hedenmalm, Håkan	VR	Reproducing kernels, flows, condensation phenomena, and conformal mapping.	1 950 000
SSF 13	Hoffman, Johan	SSF	Adaptive Computation of Turbulent Flow	3 000 000
2005-898	Hoffman, Johan	VR	Construction, analysis and implementation of adaptive methods for turbulent flow	1 782 000
2005-900	Hoppe, Jens	VR	M(atrrix)-Theory and Integrable Systems	1 782 000
2004-3687	Hultman, Axel	VR	Algebraic and topological combinatorics.	3 516 000
2005-2832	Johnson, Claes ¹	VR	New paradigm of Computational Mathematical Modeling	1 783 300
2006-3279	Jonsson, Jakob	VR	Interactions between topological combinatorics and other fields of mathematics	3 615 000
2004-3594	Jonsson, Mattias	VR	Singularities in geometry, analysis and dynamics.	1 623 000
2002-4771	Karlsson, Anders	VR	Boundaries of groups and actions on nonpositively curved spaces	3 158 064
2003-5200	Kozlov, Dmitry ¹	VR	Algebraic and Topological Combinatorics	940 500
2003-5443	Kreiss, Gunilla	VR	Hydrodynamical Stability of Shear flows	1 539 000

Royal Institute of Technology (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2004-2371	Kreiss, Gunilla	VR	Non-reflecting boundary conditions for wave propagation problems	541 000
2002-4956	Kurlberg, Pär	VR	Number theory related to Quantum Chaos	628 680
2005-949	Kurlberg, Pär	VR	Number theory related to Quantum Chaos	1 782 000
2003-5335	Landén, Camilla ¹	VR	Arbitrage pricing of derivatives on a non-ideal or non-traded underlying	1 279 800
SSF 1	Lindquist, Anders	SSF	Center for Industrial and Applied Mathematics	26 000 000
2005-929	Linusson, Svante	VR	Combinatorial Optimization Problems with Random Weights	810 000
2003-5373	Ringström, Hans	VR	Mathematical analysis of the asymptotic behaviour of solutions to the Einstein equations in the cosmological setting	3 375 200
2003-5431	Ruhe, Axel ¹	VR	Model reduction for systems with multiple scales	923 400
2004-4305	Runborg, Olof	VR	Multiscale methods for high-frequency wave propagation	1 712 400
2006-3264	Saprykina, Maria	VR	Multidimensional dynamical systems.	3 615 000
2002-4973	Shahgholian, Henrik	VR	Free boundary problems, theory and applications	419 120
2003-5341	Shahgholian, Henrik	VR	Fri boundary value problems, theory and applications	1 316 400
2006-2698	Shahgholian, Henrik	VR	Global and geometric aspects in PDE & FBP	1 950 000
2002-4965	Skjelnes, Roy	VR	The construction of parameter spaces of points and of zero dimensional cycles	3 158 064
2002-5280	Svanberg, Krister	VR	Topology optimization of load-carrying structures	2 636 400
2006-3236	Svanberg, Krister	VR	Topology optimization of stress-constrained structures	1 919 000

Royal Institute of Technology (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2002-4961	Szepessy, Anders	VR	Approximation of optimal control problems for partial differential equations	1 460 160
2005-3097	Szepessy, Anders	VR	Approximation of optimal control and hydrodynamic limits using Hamilton-Jacobi equations in infinite dimension	1 188 000
SSF 16	Wikström, Douglas	SSF	Provably Secure and Practical Cryptography	3 000 000

Linköping University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2002-5317	Asratian, Armen	VR	Some graph problems arising in telecommunication networks	1 257 360
2002-4876	Berntsson, Fredrik	VR	Cauchy Problems and Coefficient Identification for Elliptic and Parabolic PDE	2 064 348
2002-5536	Björn, Jana	VR	Non-linear partial differential equations and non-linear potential theory on Euclidean and metric spaces	3 158 064
2005-885	Eldén, Lars	VR	Numerical multilinear algebra: Theory, Algorithms and Applications	1 782 000
2003-5345	Holmberg, Kaj	VR	Design of survivable communication networks with distributed routing	1 316 400
2006-3227	Holmberg, Kaj	VR	Directing, Controlling and Modeling IP Network Traffic	1 950 000
2002-4907	Izquierdo Barrios, Milagros	VR	Studies of Riemann and Klein surfaces with combinatorial methods	1 257 360
2004-4214	Koski, Timo	VR	Probabilistic Modelling in Bioinformatics: Combination of Classifiers in Bacterial Taxonomy and 3D Protein Interaction	541 000
2002-4920	Kozlov, Vladimir	VR	An asymptotic theory for linear and nonlinear partial differential equations	1 257 360

Linköping University (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-2846	Kozlov, Vladimir	VR	Partial differential equations with measurable coefficients: an asymptotic approach	1 660 500
2005-2874	Larsson, Torbjörn	VR	Large-scale combinatorial optimization - Theoretical and algorithmic developments from new global optimality conditions	1 783 300
2002-4842	Maz'ya, Vladimir ¹	VR	Singularities in Boundary Value problems with Applications to Continuum Mechanics and Physics	419 120
2002-4850	Shaposhnikova, Tatiana ¹	VR	Brezis-Mironescu problem and applications of multipliers to boundary value problems and integral equations	1 257 360
2005-3200	Värbrand, Peter	VR	Mathematical Optimization in the Design of UMTS Networks	1 783 300
2004-3902	Yuan, Di	VR	Integer Programming for Problems Related to Power Control in Ad Hoc Networks	690 000
2005-3212	Yuan, Di	VR	Energy-Efficient Broadcasting in Mobile Ad Hoc Networks: Distributed Algorithms and Performance Simulation	789 600

Luleå University of Technology

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2004-3862	Grip, Niklas	VR	New methods and results for automatic vibration analysis, shift-invariant spaces, wavelet-based Tikhonov regularization and a wide range of applications: Spreading new theory to a broad audience.	3 605 400
2005-3168	Wall, Peter	VR	An interdisciplinary study of rough surface effects in lubrication by homogenization techniques	1 782 000

Lund University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-925	Kurasov, Pavel	VR	Quantum graphs: trace formulas and dynamical zeta-functions.	1 660 500
2004-3668	Schmeling, Jörg ¹	VR	Structural Analysis of Non-invertible Dynamical Systems with Singularities and Partial Hyperbolicity	541 000
2005-2999	Schmeling, Jörg ¹	VR	Non-hyperbolic structures in smooth dynamical systems	1 782 000
2002-5294	Sparr, Gunnar ¹	VR	Mathematical modeling and analysis of the retinal nerve fiber layer	1 460 160
2002-5375	Asmussen, Søren	VR	Pricing of options with Levy driven background processes	1 460 160
2003-5287	Constantin, Adrian ¹	VR	Mathematical studies of water wave phenomena	1 259 700
2003-5362	Dencker, Nils	VR	The solvability of partial differential operators	2 835 000
2006-2629	Dencker, Nils	VR	The Solvability of Differential Operators	1 950 000
2004-3672	Helsing, Johan	VR	Flexible and efficient integral equation methods for computational materials science	1 623 000
SSF 7	Holst, Ulla	SSF	Spatial statistics and image analysis for environment and health	9 000 000
2005-2799	Lindström, Erik	VR	Modelling and Forecasting in Non-Linear Stochastic Dynamic Systems	1 782 000
SSF 10	Roos, Björn ¹	SSF	Computational Chemistry Software for Industry and Academia	7 500 000
2004-3652	Rydén, Tobias	VR	Statistical inference in partially observed models	1 419 000
2002-4807	Schmeling, Tatiana ¹	VR	Dynamics of complex networks	1 095 120
2003-5262	Schmeling, Tatiana ¹	VR	Markov Processes on Dynamical Random Graphs	1 316 400
2002-5370	Söderlind, Gustaf	VR	Adaptive time-stepping methods for ODEs, DAEs and PDEs	1 460 160

Lund University (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-3129	Söderlind, Gustaf	VR	Adaptivity: Control, signal processing and grid density	1 782 000
2006-3213	Tajvidi, Nader	VR	Statistical inference for multi-variate extremes: methods and applications	1 906 000
2002-5415	Wiktorsson, Magnus	VR	Simulation and approximation of stochastic processes related to Lévy processes	2 752 464
2005-3230	Åström, Karl	VR	Geometry of multi-camera systems	1 782 000

Mid Sweden University²

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2002-5308	Kutzschebauch, Frank	VR	Complex analytic geometry and pluripotential theory	1 460 160

Mälardalen University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2004-3608	Eriksson, Kimmo	VR	Game theory of matchings and coalitions	1 590 000

Stockholm University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2003-5425	Britton, Tom	VR	Phylogenetics and epidemics: stochastic modelling and statistical inference	1 086 800
2006-3520	Britton, Tom	VR	Random networks, epidemics and phylogenetics	1 950 000
2005-881	Deijfen, Maria	VR	Stochastic spatial structures and random graphs	3 186 000
2005-2615	Ekedahl, Torsten	VR	Algebraic structures in algebraic geometry and homotopy theory	1 518 600
2005-2810	Hössjer, Ola	VR	Statistical modelling and probability theory with genetic applications.	1 660 500

Stockholm University (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-2891	Maad, Sara ¹	VR	Embedded eigenvalues in mathematical physics and non-linear waves	3 186 000
2004-2097	Merkulov, Sergei	VR	Operads, deformation theory and differential geometry	1 623 000
2004-3940	Palmgren, Juni ³	VR	Causal inference for controlled experiments and observational studies	1 623 000
2002-5566	Passare, Mikael	VR	Graduate school in geometry	1 460 160
2003-5203	Passare, Mikael	VR	Mathematical methods for inverse problems in tomography	546 800
2006-2676	Passare, Mikael	VR	Amoebas, discriminants, and asymptotics in one and several complex variables	1 950 000
2004-4199	Rullgård, Hans ¹	VR	Inverse and ill-posed problems with applications in electron tomography.	3 516 000
2003-5277	Shapiro, Boris	VR	Geometry of polynomials and polynomial eigenfunctions	1 316 400
2006-2701	Shapiro, Boris	VR	Stokes geometry of linear ordinary differential equations and related problems in spectral theory	1 743 000
2005-3101	Szulkin, Andrzej	VR	Topological methods in nonlinear differential equations	1 660 500
SSF 11	Palmgren, Juni ³	SSF	Development of scientific education in biostatistics and stochastic bio-information	10 000 000
SSF 6	Palmgren, Juni ³	SSF	Multivariate Statistical Models in Epidemiology, Genetics and Bioinformatics	13 500 000

Swedish University for Agricultural Sciences

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2002-5578	von Rosen, Dietrich ¹	VR	Multivariate Linear Models with Singular Covariance Matrices	1 216 800
2005-3186	von Rosen, Dietrich ¹	VR	Estimating the Rank Covariance Matrix when variables are partially uncorrelated	567 000

Umeå University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2005-2805	Häggkvist, Roland [†]	VR	Scheduling by restricted colorings	1 782 000
2004-4187	Kågström, Bo	VR	Hierarchically Blocked Algorithms and Optimized Low-Level Kernels for Dense Matrix Computations on Memory-Tiered High-Performance Computing Systems	1 712 400
SSF 5	Kågström, Bo	SSF	Matrix Pencil Computations in Computer-Aided Control System Design: Theory, Algorithms and Software Tools	11 500 000
2004-4201	Larson, Mats	VR	Automatic adaptive finite element method for problems with complicated geometry and multiscale features	1 712 400

Uppsala University

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2003-5264	Borgefors, Gunilla	VR	Digital Geometry with Applications to Volume Image Processing, DIGAVIP	1 859 100
2002-8296	Ekholm, Tobias	VR	Interactions between geometry, topology and quantum invariants - in knot theory and elsewhere	135 200
2005-2756	Holmgren, Sverker	VR	Computational Methods for High-Dimensional Problems in Quantum Molecular Dynamics	1 782 000
2002-5532	Holmgren, Sverker	VR	Flexible parallel numerical schemes for the time-dependent Schrödinger equation	1 460 160
2003-5267	Janson, Svante	VR	Partial and stochastic differential equations and financial mathematics	1 316 400
2003-5240	Juhl, Andreas [†]	VR	Harmonic analysis and dynamics on Kleinian manifolds	1 316 400
2004-4218	Kaj, Ingemar	VR	Scaling phenomena and long-range dependence in stochastic systems	1 419 000
SSF 9	Kaj, Ingemar	SSF	Stochastic methods in telecommunications	6 000 000

Uppsala University (continued)

Dnr	Main grant holder	Funding source	Project title	Funding (SEK)
2006-3537	Kreiss, Gunilla	VR	Perfectly matched absorbing layers for wave propagation problems	1 950 000
2005-2868	Larsson, Elisabeth	VR	Radial basis function methods for high-dimensional problems	1 660 500
2004-2226	Lötstedt, Per	VR	Numerical methods for problems of high dimension in molecular biology	1 379 000
2004-984	Mazorchuk, Volodymyr	VR	Stratified categories, functors, and applications	1 419 000
2004-4208	Palmgren, Erik	VR	Constructive mathematics and exact computation on higher order objects	135 000
2002-4786	Tintarev, Kyril	VR	Dirichlet forms in spaces of homogeneous type	1 257 360
2005-3152	Tucker, Warwick	VR	Auto-validating numerical methods for viscous shocks	1 782 000
2006-2728	Tysk, Johan	VR	Partial and stochastic differential equations and financial mathematics	1 839 000
2002-5441	Viro, Oleg ¹	VR	Categorification in low dimensional topology	1 460 160
2003-5280	von Sydow, Lina	VR	Adaptive high-order finite difference methods for the multi-dimensional Black-Scholes-Merton equation	1 296 000
2003-5217	Zwanzig, Silvelyn	VR	SIMEX: another look at the bootstrap	1 316 400

Footnotes:¹ no report² The only grant recipient in the period at the Mid Sweden University has now moved to Germany³ also at KI⁴ Juni Palmgren, see SU

APPENDIX 2: QUESTIONNAIRE FOR THE SELF-ASSESSMENT

Personal data

Personal data, contact and occupational information.

Area

Research Area: Choice between the different alternatives: Algebra, Analysis, Applied mathematics, Combinatorics, Discrete mathematics, Engineering mathematics, Geometry, Mathematical Physics, Mathematical Statistics, Numerical Analysis as first, second and third research area.

Executive summary: an executive summary of the research

Homepage: link(s) to homepage for further background information

Financial Support

Financial support during the period 2002-2009. Grants (greater than SEK 100 000/year) where the project leader is the grant holder listed.

Publications

Publications 2002-2009 applicable to Mathematics. The 15 most important publications, and the total number of publications.

Link can be provided to a web page where the full publication list can be found.

Personnel

PhD/Lic degrees awarded during 2002-2009 and ongoing PhD/Lic during 2002-2009.

Visiting researchers (Postdocs, Guest researchers and Professors) during 2002-2009. Name, type, project, and period of time.

Cooperation

Description of ongoing national and international cooperation. Partner, country, type and extent of cooperation and period of time. (Characters: max 2000)

Description of outreach activities related to the research

Scientific results

Summary the most significant scientific achievements and results for the period 2002-2009, (Characters: max 4000)

Research in Sweden (Mathematics)

Comments on research in Sweden: Strengths and weaknesses of Swedish research in, or close to, the rapporteur's own major area/s. (Characters: max 1500)

Future: Future plans in terms of 1) direction of research area, 2) research group, 3) national and international cooperation. (Characters: max 1000)

Infrastructure

Infrastructure investments: Description of any larger investments (>2M SEK) in infrastructure during 2002-2009 and/or further needed equipment/ infrastructure for the research.

Institut Mittag-Leffler: Description of participation/involvement in activities run by the Institut Mittag-Leffler
(Characters: max 1500)

Other

Technical feedback: Technical feedback related to the web form or its format, submission, etc.

Other information: other information related to the questionnaire

APPENDIX 3: HEARINGS

The mathematics departments were asked to come to a hearing with the expert committee in order to provide background on the environment where the project leaders were active.

They were asked to prepare a short presentation containing the following:

- The research in mathematics performed at the department; approximately what proportions of the research falls within applied mathematics, engineering mathematics or pure mathematics, and in which fields (e.g. analysis etc), as well as the most important results of the research.
- The international and national collaboration of the researchers at the department
- Applications of the research, collaborations with business/society
- Organization of the department: -- approximate numbers of PhD students, postdocs, junior researchers, senior researchers (lektor) and professors are there (in mathematics)? What is the department organization?
- The visions and future plans of the mathematics group, and if there are any planned special initiatives.
- Access to and use of infrastructure. Is there any need for new infrastructure?
- Are there any researchers or PhD students that have participated in programmes arranged by the Mittag-Leffler institute?
- The importance for the department of having institutes like the Mittag-Leffler.

Representatives from the following universities attended the hearings:

Luleå University of Technology

Karolinska Institutet

Stockholm University

KTH Royal Institute of Technology

Linköping University

Chalmers University of Technology /Gothenburg University

Umeå University

Lund University/LTH

Uppsala University

Karlstad University and Mälardalen University could not be present for a hearing, but sent a written answer to the questions. The Swedish University for Agricultural Sciences did not answer the invitation.

APPENDIX 4: SHORT CV'S OF THE MEMBERS OF THE EVALUATION COMMITTEE

Professor Ulf Danielsson (Chairman)

Department of Physics and Astronomy, Uppsala University

Web: <http://www.fysast.uu.se/teorfys/en>

Research interests: Theoretical Physics; String theory and cosmology.

Professor Kari Astala

Department of Mathematics and Statistics, University of Helsinki

Web: <http://www.helsinki.fi/facultyofscience/research/astala.html>

Research interests: different areas in mathematical analysis, from complex and harmonic analysis to partial differential equations (PDEs) and dynamical systems.

Professor Björn Birnir

Department of Mathematics, University of California

Web: <http://www.cse.ucsb.edu/people/Birnir.html>

Research interests: qualitative analysis of nonlinear PDEs; dynamical systems theory; qualitative theory of nonlinear geographic systems; nonlinear phenomena in quantum mechanical systems; and applications of the above.

Professor Geir Ellingsrud

Matematisk institutt, Universitetet i Oslo

Web: <http://www.math.uio.no/>

Research interests: algebraic geometry, and in later years also enumerative geometry with inspiration from string theory.

Professor Maria J. Esteban

CEREMADE (UMR CNRS 7534), Université Paris-Dauphine

Web: <http://www.ceremade.dauphine.fr/~esteban/>

Research interests: theory of nonlinear partial differential equations, calculus of variations, interactions fluid-structure and also in models coming from relativistic quantum mechanics and quantum chemistry.

Professor Alistair Fitt

School of Mathematics, University of Southampton

Web: <http://www.soton.ac.uk/math/people/profiles/applied/adf.html>

Research interests: current fields of research include asymptotic analysis of extensional flows (mainly connected to the production of optical fibres), studies in the flow of glass, and a range of flow and deformation problems that occur as a result of various diseases and conditions in human eyes.

Professor Roger E. Howe

Mathematics Department, Yale University

Web: http://www.math.yale.edu/public_html/People/Howe.html

Research interests: representation theory, automorphic forms, harmonic analysis, and invariant theory.

Professor Thomas G. Kurtz

Departments of Mathematics and Statistics, University of Wisconsin

Web: <http://www.math.wisc.edu/~kurtz/>

Research interests: probability theory and stochastic processes.

Professor Olavi Nevanlinna

Institute of Mathematics, Aalto University

Web: <http://math.tkk.fi/people/olavi.nevanlinna.en.html>

Research interests: numerical analysis and operator theory.

Professor Joel Spencer

Courant Institute, New York University

Web: www.cs.nyu.edu/cs/faculty/spencer/index.html

Research interests: discrete mathematics and theoretical computer science, especially the use of probabilistic techniques as pioneered by Paul Erdos.

SAMMANFATTNING PÅ SVENSKA

En internationell kommitté har utvärderat den forskning i matematik i Sverige som beviljats finansiering från Vetenskapsrådet och Stiftelsen för Strategisk Forskning under perioden 2002-2006. Resultaten presenteras i denna rapport.

Kommittén konstaterar att svensk matematik är stark inom många områden: bland annat inom matematisk analys, som har en lång historisk tradition och länge var det dominerande forskningsområdet. På senare tid har även områdena algebra och geometri växt, med intressanta tillämpningar inom fysiken. Tillämpad matematik har fått ökat genomslag under senare år, och kommittén bedömer att stora framsteg har gjorts. Numerisk analys har utvecklats till ett mycket starkt område, men ämnet räknas ofta till datavetenskapen istället för matematiken, vilket kommittén anser leder till försummade samarbetstillfällen. Vissa svaga områden där förbättringar omgående måste genomföras, exempelvis inom statistiken, noterades också. Kommittén rekommenderar vidare kompetensuppbyggnad inom områden med anknytning till biologi.

Kommittén konstaterar att arbetsvillkoren för matematiker och universitetsforskare är mycket svåra i Sverige. Systemet där finansiering från Vetenskapsrådet används för att minska undervisningsbördan anses vara mycket ineffektivt. Kommittén anser att tid till forskning istället bör garanteras av universiteten; forskare ska inte ständigt tvingas söka anslag i syfte att säkra tid att forska. Kommittén noterar också att det tycks råda svårigheter att behålla de mest framstående, unga forskarna i Sverige.

Kommittén noterade också att den effektiva anslagssumma som Vetenskapsrådet avsätter till matematik är synnerligen låg i jämförelse med andra europeiska länder och USA. Detta beror delvis på att efterfrågan på anslag för att minska den tunga undervisningsbördan slukar kapital som kunnat användas i mer kreativa syften. Även om det är svårt att göra direkta jämförelser mellan finansiering av matematik i Sverige och i andra länder, menar kommittén att den allmänna anslagsnivån är låg i förhållande till kvalitén hos de forskningsprogram som förtjänade stöd.

Kommittén var mycket bekymrad över situationen för Institut Mittag-Leffler. Det finns stor risk att denna nationella skatt går förlorad, och kommittén rekommenderar starkt att institutets kortsiktiga finansiella underskott och dess långsiktiga finansiella stabilitet ges högsta prioritet.

Kommittén imponerades av den mängd aktiviteter som riktats till gymnasielärare och gymnasister. Detta ökar inte bara kunskapen om matema-

tik och vetenskap i allmänhet, utan kan också uppmuntra gymnasister att välja matematik i sina framtida karriärer. I kontrast till detta konstateras det brister i matematikers försök att informera allmänheten om matematikens nytta i vår moderna värld. Fler svenska matematiker uppmuntras därför förmedla sitt arbete och sina resultat till allmänhet, politiker och media.

På det hela taget konstaterar kommittén att Sverige skulle dra nytta av, och borde ha råd med, mycket mer matematisk forskning än vad som i nuläget finansieras. Matematik är avgörande för utvecklingen av nya teknologier, och att investera i matematik är att investera i Sveriges framtid.

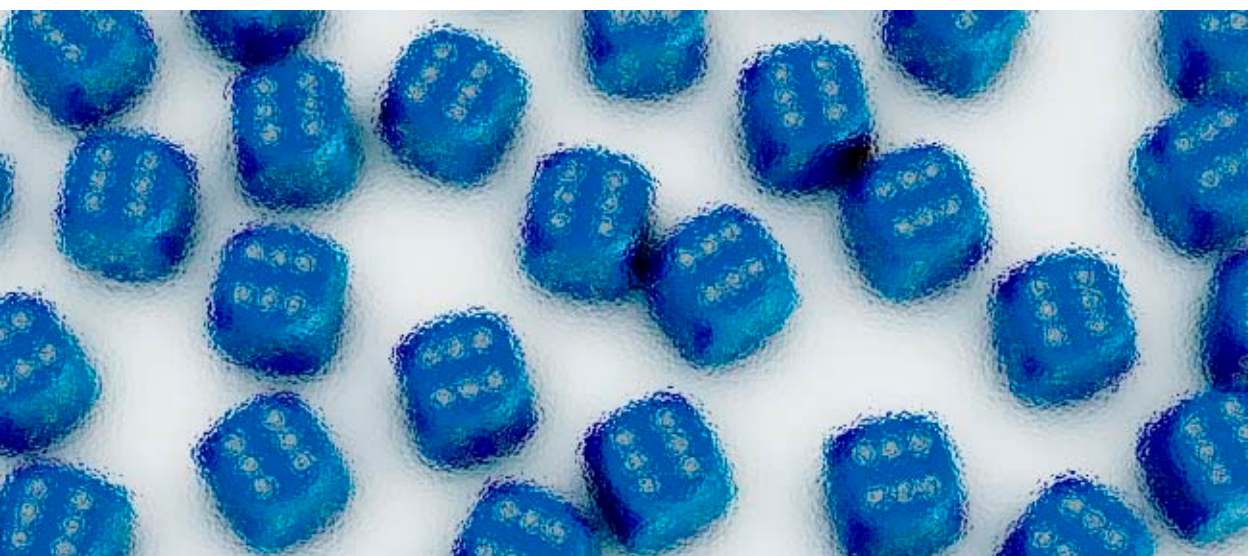
A committee has been invited to evaluate 123 projects within Swedish mathematics funded by the Swedish Research Council or the Swedish Foundation for Strategic Research during the years 2002–2006. The results are presented in this report.



Vetenskapsrådet



SWEDISH FOUNDATION *for*
STRATEGIC RESEARCH



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The Swedish Research Council is a government agency that provides funding for basic research of the highest scientific quality in all disciplinary domains. Besides research funding, the agency works with strategy, analysis, and research communication. The objective is for Sweden to be a leading research nation.

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